

# **Churchside Federation**

Mundford C of E Primary Academy

Gooderstone C of E Primary Academy

## **Mathematics Long-term Plan**

## Introduction

This plan is based on the National Curriculum Programme of Study (published September 2013) and shares the same aims, that all pupils should:

- become **fluent** in the fundamentals of mathematics, including though varied and frequent practise with increasingly complex problems, so that over time pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language.
- **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. Although organised into domains, pupils should make connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge in science and other subjects.

The expectation is that the majority of pupils will move through the programme of study at broadly the same pace. However, decisions about when to progress should always be based on the security of pupil's understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practise, before moving on.

### Mathematical Vocabulary

Children are expected to read and spell mathematical vocabulary. At Key Stage 1, this will be at a level consistent with their increasing word reading and spelling knowledge. At Key Stage 2, they should be taught to read and spell mathematical vocabulary confidently and correctly, using their growing word-reading and spelling knowledge.

### Speaking & Listening

The national curriculum for mathematics reflects the importance of spoken language development. Children should have frequent opportunities to discuss and reason about mathematics, including presenting mathematical justification, argument or proof. They must be assisted in making their thinking clear both to themselves and to others. Through questioning & discussion, teachers should model mathematical vocabulary and reasoning and probe misconceptions.

### Assessment

Teacher's use Edison/NAHT Achievement Statements (I can's) to assess pupils progress throughout the year. Termly assessments are used to check on children's retention and ability to apply skills taught.

# Problem Solving & Reasoning

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	
	<ul style="list-style-type: none"> <li>understand &amp; use in practical contexts: <i>operation, sign, number sentence.</i></li> <li>choose &amp; use appropriate number operations &amp; ways of calculating to solve problems in a wide variety of context, including other subjects and use of money &amp; measures.</li> <li>Solve one step 'story' problems.</li> <li>decide whether a calculation can be done mentally or needs the use of apparatus, such as counters, coins etc.</li> <li>Make up number 'stories' to reflect statements.</li> <li>Explain orally how a problem was solved.</li> <li>Solve simple puzzles &amp; problems using mathematical content they know.</li> <li>Give examples to match statements.</li> </ul>	<ul style="list-style-type: none"> <li>understand, use &amp; read: <i>operation, sign, symbol, number sentence.</i></li> <li>choose &amp; use appropriate number operations &amp; ways of calculating to solve problems in a wide variety of context, including other subjects and use of money &amp; measures.</li> <li>make up number stories for all four operations.</li> <li>Decide whether a calculation can be done mentally or needs the use of apparatus.</li> <li>Identify missing numbers and operations in calculations.</li> <li>Solve simple one- and two-step word problems.</li> <li>explain orally and/or record how a problem was solved.</li> <li>Solve puzzles &amp; problems using mathematical content they know.</li> <li>investigate general statements by finding examples that match it.</li> </ul>	<ul style="list-style-type: none"> <li>understand, use &amp; read: <i>operation, sign, symbol, number sentence, equation, calculation.</i></li> <li>choose &amp; use appropriate number operations &amp; ways of calculating to solve problems in a wide variety of context, including other subjects and use of money &amp; measures.</li> <li>decide whether a calculation needs to be done mentally, with apparatus or with jottings or written method; explain their methods orally and in writing.</li> <li>Solve one- and two-step word problems.</li> <li>Identify missing numbers and operations in calculations.</li> <li>Look at different calculations for the same operation and say which is hardest/easiest and why.</li> <li>solve puzzles and problems using mathematical content they know.</li> <li>investigate a general statement by finding examples that match it or disprove it.</li> </ul>	<ul style="list-style-type: none"> <li>Choose &amp; use the appropriate operation(s) to solve 2-step word problems, including use of time, money, measures, fractions and application in other subjects.</li> <li>Decide which calculations can be done mentally or with pencil &amp; paper.</li> <li>Explain &amp; record how a problem was solved, including explaining orally their mental calculation strategies.</li> <li>Make &amp; justify decisions.</li> <li>Look at a set of + or - calculations and say which is the easiest/hardest and why.</li> <li>Explain how calculations have been solved, using numbers, signs and symbols.</li> <li>Solve mathematical puzzles in a range of contexts.</li> <li>Identify missing operations, and numbers in calculations.</li> <li>Start to express a relationship in words.</li> <li>Find examples that match a general statement and begin to suggest general statements of their own.</li> </ul>	<ul style="list-style-type: none"> <li>Choose &amp; use the appropriate operation(s) to solve multi-step word problems, including use of time, money, measures, fractions and application in other subjects.</li> <li>Decide which calculations can be done mentally, with pencil &amp; paper or with written methods.</li> <li>Explain &amp; record how a problem was solved, including explaining orally their mental calculation strategies.</li> <li>Make &amp; justify decisions.</li> <li>Look at a set of multiplications - say which is the easiest/hardest and why.</li> <li>Explain how calculations have been solved, using numbers, signs and symbols.</li> <li>Solve mathematical puzzles in a range of contexts.</li> <li>Identify missing operations, and numbers in calculations.</li> <li>Express a relationship in words.</li> <li>Find examples that match a general statement; suggest &amp; test general statements of their own.</li> </ul>	<ul style="list-style-type: none"> <li>Choose &amp; use the appropriate operation(s) to solve multi-step word problems, including use of time, money, measures, fractions and application in other subjects.</li> <li>Decide which calculations can be done mentally, with pencil &amp; paper or with written methods.</li> <li>Explain &amp; record how a problem was solved, including explaining orally their mental calculation strategies.</li> <li>Make &amp; justify decisions.</li> <li>Look at a set of divisions and say which is the easiest/hardest and why.</li> <li>Explain how calculations have been solved, using numbers, signs and symbols.</li> <li>Solve mathematical puzzles in a range of contexts.</li> <li>Identify missing operations, and numbers in calculations.</li> <li>Express a relationship in words &amp; start to use simple formulae.</li> <li>Find examples that match a general statement; identify, suggest &amp; test their own general statements.</li> </ul>	
Examples	Puzzles	<p>How many different ways can you score 6 with two dice? Ann is two years older than Tom – how old could each of them be?</p>	<p>How many dominoes have an odd total number of spots? Using three dice, how many different ways can you make a total of 12?</p> <p>624 <math>\square</math> 8 = 32    94 <math>\square</math> 5 = 89 20 <math>\div</math> <math>\square</math> = 5    <math>\square</math> - 2 = 19</p>	<p>Find a pair of numbers with a sum of 7 and a product of 10.</p> <p>Use 2, 4, 5 and the signs +, x and =. How many different answers can you make between 40 and 200?</p> <p>Fit two shapes together to make a symmetrical pattern.</p>	<p>Find a pair of numbers with a sum of 15 and a product of 54.</p> <p>72 cubes can be arranged to make a 2x3x12 cuboid. What other cuboids can you make?</p> <p>19 <math>\square</math> 21 = 40    80 <math>\square</math> 6 = 480 72 - <math>\square</math> = 29    <math>\square</math> x 2 = 14</p>	<p>Find 2 consecutive numbers with a product of 182.</p> <p>Find ways to complete: <math>\square + \square + \square = 1</math></p> <p>How many different rectangles can you make from 12 squares?</p>	<p>For how many 3-digit numbers does the sum of the digits equal 25? Replace each <math>\blacklozenge</math> with one of the digits from 1 to 6, to make this correct: <math>\blacklozenge \times \blacklozenge = \blacklozenge \blacklozenge</math>.</p> <p>Using straight cuts, divide a square into 6 smaller squares</p>
	General Statements	<p>When I add 10 to a number, the units number stays the same.</p> <p>All triangles have 3 sides.</p> <p>I can add numbers in any order &amp; the answer is the same.</p>	<p>Odd numbers have a remainder of 1 when divided by 2.</p> <p>A cube has six square faces.</p> <p>If a number ends in 0, it can be divided exactly by ten.</p>	<p>If I multiply numbers in any order, you get the same answer.</p> <p>A multiple of 5 is always a multiple of 10.</p> <p>A square has 4 right-angles.</p>	<p>If I multiply by 10, every digit moves one place left.</p> <p>The perimeter of a rectangle is 2x the length plus 2x the width.</p> <p>Explain how to find the number of days in any number of weeks.</p>	<p>The product of 2 consecutive numbers is even.</p> <p>Angles on a straight line add up to 180 degrees.</p> <p>A sequence starts 1, 4, 9, 16, 25. Explain the rule in words</p>	<p>If you add 3 consecutive numbers, the sum is 3x the middle number.</p> <p>Use symbols to write a formula for the number of months <math>m</math> in <math>y</math> years.</p> <p>Write a formula for the <math>n</math>th term in a sequence.</p>

# Developing Algebraic Thinking

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
<ul style="list-style-type: none"> <li>describe simple patterns and relationships involving numbers or shapes</li> <li>recognise, create &amp; continue a repeating pattern;</li> <li>understanding equals as a balance &amp; use to show equivalence between two number statements; understand why it doesn't just mean "this is the answer"</li> <li>sequence events in chronological order using language [for example, before and after, next, first, today, yesterday, tomorrow, morning, afternoon and evening]</li> <li>recognise a symbol such as  to represent a missing number</li> <li>solve missing number problems</li> </ul>	<ul style="list-style-type: none"> <li>recognise a symbol such as  to represent a missing number</li> <li>find an unknown number in a number sentence, using the symbols +, -, x, ÷ and =.</li> <li>understand the = as equality and use to show equivalence between two statements or number sentences.</li> <li>solve problems and puzzles, working in an organised way and explaining their methods in pictures, words or orally.</li> <li>solve puzzles where there is more than one answer</li> <li>describe patterns, recognise simple generalisations and predict what will come next.</li> </ul>	<ul style="list-style-type: none"> <li>Understand and use the equal sign as the balance of an equation</li> <li>Recognise symbols/letters can represent numbers</li> <li>Solve missing number &amp; shape problems</li> <li>Solve puzzles where there is more than one answer (<i>key strategy: another, another another...</i>)</li> <li>Solve problems that lead to generalisations and notice patterns</li> </ul>	<ul style="list-style-type: none"> <li>state inequalities using the symbols &lt; and &gt; (e.g. <math>-3 &gt; -5</math>, <math>-1 &lt; +1</math>)</li> <li>Represent puzzles or problems using numbers sentences, using +, -, x, ÷ and =, as well as symbols or empty boxes to represent unknowns.</li> <li>Solve missing number problems.</li> <li>Solve problems where there is more than on answer. (<i>key strategy: another, another another...</i>)</li> </ul> <p>Notice patterns and make generalisations.</p>	<ul style="list-style-type: none"> <li>Pupils use and explain the = sign to indicate, equivalence, including missing number problems (for example, <math>13 + 24 = 12 + 25</math>; <math>33 = 5 \times \square</math>)</li> <li>Use the relations of perimeter or area to find unknown lengths.</li> <li>Use simple algebra to express missing measurements (e.g. <math>4s = 24</math> for a square with a perimeter or 24cm and missing sides)</li> <li>Solve equations with missing numbers</li> <li>Understand what letters represent in algebraic expressions</li> <li>Make and investigate a general statement about familiar numbers by finding examples that satisfy it. Explain a generalised relationship (formula) in words</li> </ul>	<ul style="list-style-type: none"> <li>express missing number problems algebraically</li> <li>represent a real life situation using algebra</li> <li>rearrange and simplify expressions</li> <li>manipulate an equation to find a solution</li> <li>use simple formula expressed in words</li> <li>generate &amp; describe linear number sequences; generate the nth term</li> <li>find pairs of numbers that satisfy number sentences involving two unknowns</li> <li>enumerate all possibilities of combinations of two variables</li> </ul>

Number and place value	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	<ul style="list-style-type: none"> <li>•count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number</li> <li>•count, read and write numbers to 100 in numerals; count in multiples of twos, fives and tens</li> <li>•given a number, identify one more and one less</li> <li>•identify and represent numbers using objects and pictorial representations including the number line, &amp; use language of: equal to, more than, less than (fewer), most, least</li> <li>•read and write numbers from 1 to 20 in numerals and words</li> <li>•recognise the place value of each digit in a two digit number</li> <li>•partition a two digit number into a multiple of tens and ones</li> <li>•ten more or ten less than any given two-digit number</li> <li>•give a reasonable estimate of a number of objects and then count them</li> </ul>	<ul style="list-style-type: none"> <li>•count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward</li> <li>•recognise the place value of each digit in a two-digit number</li> <li>•compare and order numbers from 0 up to 100; use &lt;, &gt; and = signs</li> <li>•identify, represent and estimate numbers using different representations, including the number line</li> <li>•read and write numbers to at least 100 in numerals and in words</li> <li>•use place value and number facts to solve problems</li> </ul>	<ul style="list-style-type: none"> <li>•count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number</li> <li>•recognise the place value of each digit in a three-digit number</li> <li>•compare and order numbers up to 1000</li> <li>•identify, represent and estimate numbers using different representations</li> <li>•read and write numbers up to 1000 in numerals and in words</li> <li>•solve number and practical problems involving these ideas</li> </ul>	<ul style="list-style-type: none"> <li>•count in multiples of 6, 7, 9, 25 and 1000</li> <li>•find 1000 more or less than</li> <li>•count backwards through zero to include negative numbers</li> <li>•recognise the place value of each digit in a four-digit number</li> <li>•order and compare numbers beyond 1000</li> <li>•identify, represent and estimate numbers using different representations</li> <li>•round any number to the nearest 10, 100 or 1000</li> <li>•solve number and practical problems that involve all of the above</li> <li>•read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value</li> </ul>	<ul style="list-style-type: none"> <li>•count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000</li> <li>•interpret negative numbers in context, count forwards &amp; backwards with positive and negative whole numbers, including through zero</li> <li>•read, write, order and compare numbers up to 1 000 000 and determine the value of each digit</li> <li>•round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000</li> <li>•solve number and practical problems that involve all of the above</li> <li>•read Roman numerals to 1000 (M) and recognise years written in Roman numerals</li> </ul>	<ul style="list-style-type: none"> <li>•use negative numbers in context, and calculate intervals across zero</li> <li>•read, write, order and compare numbers up to 10 000 000 and determine the value of each digit</li> <li>•round any whole number to a required degree of accuracy</li> <li>•solve number and practical problems that involve all of the above</li> </ul>
Notes & Guidance	Objects & pictorial representations should be used to support understanding. Children should recognise & create patterns with objects & shapes.	Include partitioning numbers in different ways, e.g. - 23=20+3 - 23=10+13 - 20+3=23 etc	Include partitioning numbers in different ways, e.g. - 234 = 200+30+4 - 234 = 100 + 130 + 4 - 234 = 230+4 etc	NB: place value, counting & rounding of decimal numbers is included under the Proportion strand. However, it may be appropriate to make links when teaching these skills with whole numbers.		
The above objectives set out the expectations for what children of each year group should achieve independently. However, across all years, children need exposure to and modelling of numbers that go beyond those set out for their year group.						
Models & Images	Number lines, number tracks, Numicon, 100 square, counters, objects, balance scales (to illustrate the meaning of equals).	Children should be able to represent 2-digit numbers with: Numicon, arrow cards, base ten apparatus, coins. Number lines, number tracks Balance scales	Number lines, 100 squares (including those not starting from 1), arrow cards, place value apparatus, Numicon, coins.	Number lines, 100 squares (including those starting with numbers other than 1), arrow cards, place value apparatus, coins, thermometers (negative numbers), place value charts	Number lines, 100 squares (including those starting with numbers other than 1), arrow cards, place value apparatus, coins, thermometers (negative numbers), place value charts	Number lines, 100 squares (including those starting with numbers other than 1), arrow cards, place value apparatus, coins, thermometers (negative numbers), place value charts

<b>Proportion</b> (inc. Fractions, Decimals, Percentages & Ratio)	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	
	<ul style="list-style-type: none"> <li>•count up and down in halves and quarters</li> <li>•recognise, find and name a half as one of two equal parts of an object, shape or quantity</li> <li>•recognise, find and name a quarter as one of four equal parts of an object, shape or quantity</li> <li>• Pupils connect halves and quarters to the equal sharing and grouping of sets of objects and to measures.</li> <li>• recognising and combining halves and quarters as parts of a whole.</li> </ul>	<ul style="list-style-type: none"> <li>•count up and down in halves, quarters &amp; thirds.</li> <li>•recognise, find, name and write fractions <math>\frac{1}{3}</math>, <math>\frac{1}{4}</math>, <math>\frac{2}{4}</math> and <math>\frac{3}{4}</math> of a length, shape, set of objects or quantity</li> <li>•write simple fractions for example, <math>\frac{1}{2}</math> of 6 = 3 and recognise the equivalence of <math>\frac{2}{4}</math> and <math>\frac{1}{2}</math>.</li> </ul>	<ul style="list-style-type: none"> <li>•count up and down in tenths;</li> <li>•recognise that tenths arise from dividing an object into 10 equal parts &amp; in dividing 1-digit numbers or quantities by 10</li> <li>•compare and order unit fractions, and fractions with the same denominators</li> <li>•recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators</li> <li>•recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators</li> <li>•recognise and show, using diagrams, equivalent fractions with small denominators</li> <li>•add and subtract fractions with the same denominator within one whole [for example, <math>\frac{5}{7} + \frac{1}{7} = \frac{6}{7}</math>]</li> <li>•solve problems involving all of the above</li> </ul>	<ul style="list-style-type: none"> <li>•count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten</li> <li>•recognise and show, using diagrams, families of common equivalent fractions</li> <li>•solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number</li> <li>•add and subtract fractions with the same denominator</li> <li>•recognise and write decimal equivalents of any number of tenths or hundredths</li> <li>•recognise and write decimal equivalents to <math>\frac{1}{4}</math>, <math>\frac{1}{2}</math> and <math>\frac{3}{4}</math></li> <li>•find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths</li> <li>•round decimals with one decimal place to the nearest whole number</li> <li>•compare numbers with the same number of decimal places up to two decimal places</li> <li>•solve simple measure &amp; money problems involving fractions &amp; decimals to 2d.p.</li> </ul>	<ul style="list-style-type: none"> <li>•recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements <math>&gt; 1</math> as a mixed number</li> <li>•compare and order fractions whose denominators are all multiples of the same number</li> <li>•identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths</li> <li>•add and subtract fractions with the same denominator and denominators that are multiples of the same number</li> <li>•multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams</li> <li>•read and write decimal numbers as fractions</li> <li>•recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents</li> <li>•round decimals with two decimal places to the nearest whole number and to one decimal place</li> <li>•read, write, order and compare numbers with up to three decimal places</li> <li>•recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal</li> <li>•solve problems involving number up to three decimal places</li> <li>•solve problems which require knowing percentage &amp; decimal equivalents <math>\frac{1}{2}</math> <math>\frac{1}{4}</math> <math>\frac{1}{5}</math> <math>\frac{2}{5}</math> <math>\frac{4}{5}</math> &amp; those with a denominator of 10 or 25.</li> </ul>	<ul style="list-style-type: none"> <li>•use common factors to simplify fractions; use common multiples to express fractions in the same denomination</li> <li>•compare and order fractions, including fractions <math>&gt; 1</math></li> <li>•add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions</li> <li>•multiply simple pairs of proper fractions, writing the answer in its simplest form</li> <li>•divide proper fractions by whole numbers</li> <li>•associate a fraction with division and calculate decimal fraction equivalents (e.g. 0.375) for a simple fraction (e.g. <math>\frac{3}{8}</math>)</li> <li>•multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places</li> <li>•multiply one-digit number with up to two decimal places by whole numbers</li> <li>•use written division methods in cases where the answer has up to two decimal places</li> <li>•solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison</li> <li>•solve problems which require answers to be rounded to specified degrees of accuracy</li> <li>•recall and use equivalences between simple fractions, decimals and percentages, including in different contexts.</li> <li>• solve problems involving the relative sizes of two quantities where missing value can be found by using integer multiplication and division facts</li> <li>•solve problems involving similar shapes where the scale factor is known or can be found</li> <li>•solve problems using unequal sharing and grouping using knowledge of fractions &amp; multiples</li> </ul>	
<b>Notes &amp; Guidance</b>	Children should encounter a range of different representations of fractions (see below).			<i>At all levels, concepts should be modelled visually and with concrete apparatus before children are expected to use abstract rules. More able children can still be challenged when working with diagrams and apparatus by encouraging them to consider how they can prove or explain mathematical ideas.</i>			
Pupils should be taught throughout that percentages, decimals & fractions are different ways of expression proportions of a whole.							
<b>Models &amp; Images</b>	Circles (cakes/pizzas etc.), squares, rectangles & other shapes (e.g. exploring which can/cannot be easily folded into particular fractions) Paper number lines (can be folded to demonstrate link between fraction of a shape & of a number) Number lines marked in $\frac{1}{2}$ s, $\frac{1}{4}$ s etc. Fractions of sets of objects Numicon, unifix towers (e.g. which shapes are half/quarter/third of another) Measuring scales Fraction cubes & fraction wall			See years 1-3, plus: place value apparatus; arrow cards for decimals; place value chart; money blank 100 squares	Diagrams showing fractions of different shapes Fractions of quantities, sets & real objects Fractions & decimals as a position on a number line Fractions as comparative proportions (e.g. unifix, Numicon) Blank 100 grids for comparing fractions, decimals & percentages Place value charts, apparatus & arrow cards Measuring scales, weights (e.g. 1g, 10g, 100g, 1kg), coins, litre jugs, metre sticks etc. Fractions strips Fraction wall & fraction cubes		

<b>Calculations: Addition &amp; Subtraction</b>	<b>Year 1</b>	<b>Year 2</b>	<b>Year 3</b>	<b>Year 4</b>	<b>Year 5</b>	<b>Year 6</b>
<b>Notes &amp; Guidance</b>	Children should understand the meaning of the equals sign as ‘balancing’ (not simply answer) and relate it to balanced scales.	Children should apply their increasing knowledge of mental & written methods as set out in the calculation policy.	Children should use place value apparatus, arrow cards & partitioning to add & subtract numbers in columns. Number lines can be used to model informal & mental methods and to find differences. Formal methods should only be introduced when children are ready. <i>See calculations policy for further detail.</i>	Formal & column methods should be introduced with suitable models, images & apparatus (as set out in calculations policy). Where children are not confident with more formal methods, they should return to informal or expanded methods as appropriate until suitable understanding is reached.		
<b>Models &amp; Images</b>	real objects; numbered, then blank number line; place value apparatus; unifix; base ten; place value arrow cards; place value charts; hundred squares (including those with starting points other than 1); balance scales; Numicon.  <i>see calculations policy for further guidance</i>					

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
<b>Calculations: Multiplication &amp; Division</b>	<ul style="list-style-type: none"> <li>•make connections between multiplication &amp; number patterns, counting in 2s, 5s and 10s and arrays.</li> <li>•understand division as both grouping &amp; sharing, modelling with concrete objects</li> <li>•solve one-step problems involving multiplication &amp; division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher</li> <li>•grouping and sharing small quantities</li> <li>•count in multiples of twos, fives and tens.</li> <li>•doubling and halving</li> <li>•working with arrays helps pupils to become aware of the commutative property of multiplication, that <math>2 \times 5</math> is equivalent to <math>5 \times 2</math>.</li> <li>•recognising that multiplication and division are linked</li> </ul>	<ul style="list-style-type: none"> <li>•recall and use multiplication &amp; division facts for the 2, 5, 10 times tables</li> <li>•recognise odd &amp; even numbers</li> <li>•calculate mathematical statements for multiplication &amp; division within the multiplication tables &amp; write them using the symbols <math>\times</math>, <math>\div</math> &amp; <math>=</math></li> <li>•show that multiplication can be done in any order (commutative) and division cannot</li> <li>•solve problems involving multiplication &amp; division, using materials, arrays, repeated addition, mental methods &amp; recall of multiplication &amp; division facts, including problems in context</li> </ul>	<ul style="list-style-type: none"> <li>•recall &amp; use multiplication &amp; division facts for the 3, 4 &amp; 8 times tables</li> <li>•write &amp; calculate mathematical statements for multiplication &amp; division using the multiplication facts they know</li> <li>•multiply 2-digit numbers by 1-digit numbers, using mental &amp; informal methods &amp; progressing to formal methods in line with calculations policy</li> <li>•solve problems involving multiplying &amp; dividing including using the distributive law to multiply 2-digit numbers by 1-digit, integer scaling problems and harder correspondence problems in such as n objects are connected to m objects</li> </ul>	<ul style="list-style-type: none"> <li>•recall multiplication &amp; division facts up to <math>10 \times 10</math> and use these to derive quickly to at least <math>12 \times 12</math>.</li> <li>•use place value, known &amp; derived facts to multiply &amp; divide mentally including:               <ul style="list-style-type: none"> <li>- <math>\times</math> by 0 &amp; 1</li> <li>- <math>\div</math> by 1</li> <li>- <i>multiplying 3 numbers</i></li> </ul> </li> <li>•recognise &amp; use factor pairs and commutatively in mental calculations</li> <li>•multiply 2-digit and 3-digit numbers by a one digit number, progressing to formal methods in line with calculations policy</li> <li>•solve problems involving multiplying &amp; adding using the distributive law to multiply 2-digit numbers by 1-digit, integer scaling problems and harder correspondence problems in such as n objects are connected to m objects</li> </ul>	<ul style="list-style-type: none"> <li>•identify multiples &amp; factors, including finding all factor pairs of a number &amp; common factors of two numbers</li> <li>•solve problems involving multiplication &amp; division where larger numbers are used by decomposing them into their factors</li> <li>•know &amp; use the vocabulary of prime numbers, prime factors &amp; composite (non-prime) numbers</li> <li>•establish whether a number up to 100 is prime and recall prime numbers up to 19</li> <li>•multiply numbers up to 4-digits by a 1- or 2-digit number, using written methods (see calculations policy)</li> <li>•multiply &amp; divide numbers mentally drawing upon known facts</li> <li>•multiply &amp; divide whole numbers by 10, 100 &amp; 1000</li> <li>•divide numbers up to 4-digits by a 1-digit numbers using written methods (see calculations policy)</li> <li>•recognise &amp; use square &amp; cubed number and the notation <math>^2</math> and <math>^3</math>.</li> <li>•solve problems involving all four operations &amp; combinations of these, including understanding the meaning of the equals sign.</li> <li>•solve problems involving multiplication &amp; division, including scaling y simple fractions &amp; problems involving simple rates.</li> </ul>	<ul style="list-style-type: none"> <li>•perform mental calculations, including with mixed operations &amp; large numbers</li> <li>•multiply numbers up to 4-digits by a two-digit number using written methods (see calculations policy)</li> <li>•divide whole numbers up to 4-digits by a 2-digit number, using written methods (see calculations policy)</li> <li>•interpret remainders as whole number remainders, fractions or by rounding as appropriate for the context</li> <li>•identify common factors, common multiples &amp; prime numbers</li> <li>•use their knowledge of the order of operations to carry out calculations involving the four operations</li> <li>•solve problems involving addition, subtraction, multiplication &amp; division</li> <li>•use estimation to check answers to calculations &amp; determine, in the context of a problem, levels of accuracy</li> </ul>
Notes & Guidance	Refer to the calculations policy for more exemplification of introducing multiplication & division.	Children should apply their increasing knowledge of mental & written methods as set out in the calculation policy.	Children should use place value apparatus, arrow cards & partitioning to add & subtract numbers in columns. Number lines can be used to model informal & mental methods and to find differences. Formal methods should only be introduced when children are ready. <i>See calculations policy for further detail.</i>	Formal & column methods should be introduced with suitable models, images & apparatus (as set out in calculations policy). Where children are not confident with more formal methods, they should return to informal or expanded methods as appropriate until suitable understanding is reached.		
Models & Images	real objects; arrays; bead strings & number lines; Numicon; balance scales; place value apparatus  <i>see calculations policy for further detail.</i>					

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
<b>Measurement</b>	<p>Compare, describe and solve practical problems for:</p> <ul style="list-style-type: none"> <li>•length/height (long/short, longer/shorter, tall/short, double, half)</li> <li>•weight/mass (heavy/light, heavier than, lighter than)</li> <li>•capacity/volume (full/empty, more than, less than, quarter)</li> <li>•time (quicker, slower, earlier, later)</li> <li>•measure and begin to record length/height, weight/mass, capacity/volume &amp; time (hours, minutes, seconds)</li> <li>•measure &amp; record temperature</li> <li>•make a reasonable estimation before measuring</li> </ul> <p>•recognise and know the value of different denominations of coins and notes.</p> <p>•find totals and change for amounts up to 20p</p> <p>•sequence events in chronological order using language such as: before, after, next, first, today, tomorrow, morning, afternoon, evening.</p> <p>•recognise and use language relating to dates, including days of the week, weeks, months and years</p> <p>•tell the time to the hour and half past the hour and draw the hands on a clock face to show these times</p>	<ul style="list-style-type: none"> <li>•choose and use appropriate standard units to estimate and measure length/height (m/cm); mass (kg/g); temperature (°C); capacity (litres/ml) to the nearest appropriate unit, using rulers, scales, thermometers and measuring vessels</li> <li>•compare &amp; order lengths, mass, volume/capacity and record the results using &gt;, &lt; and =</li> <li>•recognise and use symbols for pounds (£) and pence (p); combine amounts to make a particular value</li> <li>•find different combinations of coins that equal the same amounts of money</li> <li>•solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change</li> <li>•compare and sequence intervals of time</li> <li>•tell and write the time to five minutes, including quarter past/to the hour and draw the hands on a clock face to show these times</li> <li>•know the number of minutes in an hour and the number of hours in a day</li> </ul>	<ul style="list-style-type: none"> <li>•measure, compare, add and subtract: lengths (m/cm/mm); mass (kg/g); volume/capacity (l/ml); temperature (°C);</li> <li>•measure the perimeter of simple 2-D shapes</li> <li>•add and subtract amounts of money to give change, using both £ and p in practical contexts</li> <li>•tell and write the time from an analogue clock, including using Roman numerals from I to XII, and 12-hour and 24-hour clocks</li> <li>•estimate and read time with increasing accuracy to the nearest minute; record and compare time in terms of seconds, minutes and hours; use vocabulary such as o'clock, a.m./p.m., morning, afternoon, noon and midnight</li> <li>•know the number of seconds in a minute and the number of days in each month, year and leap year</li> <li>•compare durations of events, for example to calculate the time taken by particular events or tasks.</li> </ul>	<ul style="list-style-type: none"> <li>•convert between different units of measure (e.g. Hours to minutes, km to m)</li> <li>•estimate, compare and calculate different measures, including money in pounds and pence</li> <li>•measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres</li> <li>•find the area of rectilinear shapes by counting squares</li> <li>•relate area to arrays and multiplication</li> <li>•read, write and convert time between analogue and digital 12- and 24-hour clocks</li> <li>•solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days</li> </ul>	<ul style="list-style-type: none"> <li>•convert between different units of metric measure</li> <li>•understand and use approximate equivalences between metric units and common imperial units such as inches, pounds and pints</li> <li>•estimate volume (e.g. using 1cm<sup>3</sup> blocks to build cubes &amp; cuboids) and capacity (e.g. using water)</li> <li>•measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres</li> <li>•calculate and compare the area of rectangles (including squares), and including using standard units, square centimetres (cm<sup>2</sup>) and square metres (m<sup>2</sup>) and estimate the area of irregular shapes</li> <li>•relate area to arrays and multiplication</li> <li>•solve problems involving converting between units of time</li> <li>•use all four operations to solve problems involving measure [for example, length, mass, volume, money] using decimal notation, including scaling &amp; conversion between units</li> </ul>	<ul style="list-style-type: none"> <li>•solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate</li> <li>•use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to three decimal places</li> <li>•convert between miles and kilometres</li> <li>•recognise that shapes with the same areas can have different perimeters and vice versa</li> <li>•recognise when it is possible to use formulae for area and volume of shapes</li> <li>•calculate the area of parallelograms and triangles</li> <li>•calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm<sup>3</sup>) and cubic metres (m<sup>3</sup>), and extending to other units.</li> </ul>
Notes & Guidance	The 2014 Curriculum distinguishes between volume & capacity in the following way: capacity refers to liquid volumes as measured in litres, millilitres etc.; volume refers to the amount of space a 3-dimensional shape occupies, measured in cubic units such as m <sup>3</sup> or cm <sup>3</sup> .					
	Mass & weight, volume & capacity are used changeably at this stage. Pupils move from using non-standard units to common standard units.	Comparing measures includes simple multiples such as 'half as high' and 'twice as wide'.	Compare measures using simple scaling (e.g. 5x as high) & connect to multiplication. Record using mixed units (e.g. 1kg and 200g) and simple equivalences (e.g. ½ m = 50cm)	Pupils build on their understanding of place value & decimal notation to record metric measures.	Pupils use their knowledge of place value and multiplication & division to convert between standard units.	Connect conversion to graphical representation as preparation for line graphs. Introduce compound units (such as mph) and apply their knowledge in other subjects (e.g. science).
Models & Images	clock faces; sequencing charts; money;	clock faces; money; rulers, metres sticks, measuring	As for Year 2, plus: place value charts for money/measures;	multiplication arrays & grids; rectangular shapes drawn on square grids; place value charts for money/measures;	As Year 4/5, plus: conversion line graphs; measuring apparatus	

	rulers, metres sticks, measuring jugs, weighing scales, balance scales etc.	jugs, weighing scales, balance scales etc.; thermometers/number lines with negative no's.	string (for perimeter & comparing lengths) ; analogue & digital clocks	money, rulers, metres sticks, measuring jugs, weighing scales thermometers etc.; analogue & digital clocks, calendars; 1cm <sup>3</sup> blocks (& other place value apparatus).	labelled with imperial & metric units.	
<b>Geometry: Properties of Shape</b>	<b>Year 1</b>	<b>Year 2</b>	<b>Year 3</b>	<b>Year 4</b>	<b>Year 5</b>	<b>Year 6</b>
	<ul style="list-style-type: none"> <li>recognise and name common 2-D shapes, including: squares, rectangles*, circles, triangles)</li> <li>recognise and name common 3-D shapes, including: cubes, cuboids*, pyramids &amp; spheres)</li> <li>sort shapes &amp; talk about simple properties (e.g. edges, faces and vertices)</li> <li>recognise shapes in different orientations and sizes.</li> <li>know that rectangles, triangles, cuboids and pyramids are not always similar to each other</li> <li>use everyday language to describe features of familiar 3D and 2D shapes, referring to properties such as number of faces and number of corners.</li> </ul>	<ul style="list-style-type: none"> <li>identify and describe the properties of 2-D shapes, including the number of sides and line symmetry in a vertical line.</li> <li>compare and sort common 2-D and 3-D shapes and everyday objects.</li> <li>identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces</li> <li>identify 2-D shapes on the surface of 3-D shapes.</li> <li>compare and sort common 2-D and 3-D shapes and everyday objects.</li> </ul>	<ul style="list-style-type: none"> <li>draw 2-D shapes</li> <li>make 3-D shapes using modelling materials</li> <li>recognise 3-D shapes in different orientations and describe them</li> <li>recognise angles as a property of shape or a description of a turn</li> <li>identify right angles</li> <li>identify whether angles are greater or less than right angle</li> <li>identify horizontal and vertical lines and pairs of perpendicular and parallel lines</li> </ul>	<ul style="list-style-type: none"> <li>compare and classify geometric shapes, including quadrilaterals and triangles, based on properties and sizes</li> <li>identify lines of symmetry in 2-D shapes presented in different orientations</li> <li>complete a simple symmetric figure with respect to a specific line of symmetry.</li> <li>identify acute and obtuse angles and compare and order angles up to two right angles by size</li> </ul>	<ul style="list-style-type: none"> <li>use the properties of rectangles to deduce related facts and find missing lengths and angles</li> <li>distinguish between regular and irregular polygons based on reasoning about equal sides and angles.</li> <li>identify 3-D shapes, including cubes and other cuboids, from 2-D representations</li> <li>know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles</li> <li>draw given angles, and measure them in degrees (°)</li> <li>identify: angles at a point and one whole turn (total 360°); at a point on a straight line and ½ a turn (total 180°); other multiples of 90°</li> </ul>	<ul style="list-style-type: none"> <li>illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius</li> <li>draw 2-D shapes using given dimensions and angles</li> <li>compare and classify geometric shapes based on their properties and sizes</li> <li>recognise, describe and build simple 3-D shapes, including making nets</li> <li>find unknown angles in any triangles, quadrilaterals, and regular polygons</li> <li>recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles</li> </ul>
<b>Geometry: Position &amp; Direction</b>	<ul style="list-style-type: none"> <li>describe position, direction and movement, including whole, half, quarter and three-quarter turns.</li> <li>use ordinal numbers, ie first, second, third etc.</li> <li>understand that objects can turn around a point or about a line</li> </ul>	<ul style="list-style-type: none"> <li>order and arrange combinations of mathematical objects in patterns and sequences.</li> <li>use mathematical vocabulary to describe position, direction and movement, including movement in a straight line and distinguishing between rotation as a turn and in terms of right angles for quarter, half and ¾ turns (clockwise &amp; anti-clockwise)</li> </ul>	<ul style="list-style-type: none"> <li>relate right angles to turns: two right angles make a half-turn, three make three quarters of a turn and four a complete turn</li> <li>use mathematical vocabulary to describe movement about a grid, including straight line movements &amp; rotations</li> </ul>	<ul style="list-style-type: none"> <li>describe positions on a 2-D grid as coordinates in the first quadrant</li> <li>describe movements between positions as translations of a given unit to the left/right and up/down</li> <li>plot specified points and draw sides to complete a given polygon</li> </ul>	<ul style="list-style-type: none"> <li>identify, describe and represent the position of a shape following a reflection or translation, using the appropriate language, and know that the shape has not changed</li> <li>describe positions on a 2-D grid as coordinates in at least the first quadrant</li> </ul>	<ul style="list-style-type: none"> <li>describe positions on the full coordinate grid (all four quadrants)</li> <li>draw and translate simple shapes on the coordinate plane, and reflect them in the axes</li> </ul>
<b>Notes &amp; Guidance</b>	<i>* a square is a special type of rectangle; a cube is a special type of cuboid.</i> Children should discuss similarities & differences between shapes.	Children should have opportunities to build shapes using a range of equipment & mediums. Children should discuss similarities & differences between shapes.		Co-ordinate & grid work can be related to maps.		
<b>Models &amp; Images</b>	2D & 3D shapes; Sorting hoops & diagrams; Polygon & other construction equipment; Logo & robot programmes, e.g. Turtle, Bee-bot;		<i>As for KS1, plus:</i> right-angle 'eaters'; split pin angles.	2D & 3D shapes, Polygon & other construction equipment; right-angle 'eaters', split pin angles, protractors; control programmes, such as Turtle; co-ordinate grids, maps.		

Statistics	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	<ul style="list-style-type: none"> <li>•collect data in simple lists, tally charts &amp; tables</li> <li>•construct simple pictograms or block diagrams (with 1:1 representation)</li> <li>•ask and answer simple questions about data they have collected</li> </ul>	<ul style="list-style-type: none"> <li>•interpret and construct simple pictograms, tally charts, block diagrams and simple tables</li> <li>•ask and answer simple questions by counting the number of objects in each category and sorting the categories by quantity</li> <li>•ask and answer questions about totalling and comparing categorical data</li> </ul>	<ul style="list-style-type: none"> <li>•interpret and present data using bar charts, pictograms and tables</li> <li>•solve one-step and two-step questions [for example, ‘How many more?’ and ‘How many fewer?’] using information presented in scaled bar charts and pictograms and tables</li> </ul>	<ul style="list-style-type: none"> <li>•interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graph</li> <li>•solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs</li> </ul>	<ul style="list-style-type: none"> <li>•complete, read and interpret information in tables, including timetables</li> <li>•solve comparison, sum and difference problems using information presented in a line graph</li> </ul>	<ul style="list-style-type: none"> <li>•interpret and construct pie charts and line graphs and use these to solve problems</li> <li>• calculate and interpret the mean as an average</li> </ul>
Notes & Guidance	Use cubes, post-it notes, objects etc. to build ‘real’ charts, before moving on to representing these graphically.		Use simple scales on charts (e.g. 2, 5, 10)	Use a greater range of scales	Children should still have opportunities to use other types of graph (e.g. bar charts, pictograms, tall charts) and should be taught to choose the most appropriate graphs for different purposes (link to other subjects such as science).	
		Children should be introduced to many to one correspondence in pictograms with simple ratios (2, 5, 10)			Pupils connect their work on co-ordinates & scales to their interpretation of graphs.	Pupils should connect their work on angles fraction s& percentages to pie charts.
Models & Images	Computer programmes should be used to model how data sets can be represented in different forms; allowing discussion of the similarities and differences between different graphical representations of data as well as the relative advantages and disadvantages.					
	‘real’ graphs (e.g. cubes, people, post-it notes, objects); square paper, block graphs, pictograms	square paper; relate axes on graphs to number lines; scales with different intervals; bar charts & pictograms simple line graphs (showing change over time)			square & graph paper; line graphs; pie charts;	
Venn & Carroll diagrams are no-longer statutory within the KS1 and KS2 programmes of study. However, children should still have opportunities to use these both within Maths and other subjects.						