# Calculation Policy for Mathematics

The following calculation policy has been devised to meet requirements of the National Curriculum 2014 for the teaching and learning of mathematics, and is also designed to give pupils a consistent and smooth progression of learning in calculation across the federation.

**Age Stage Expectations:**

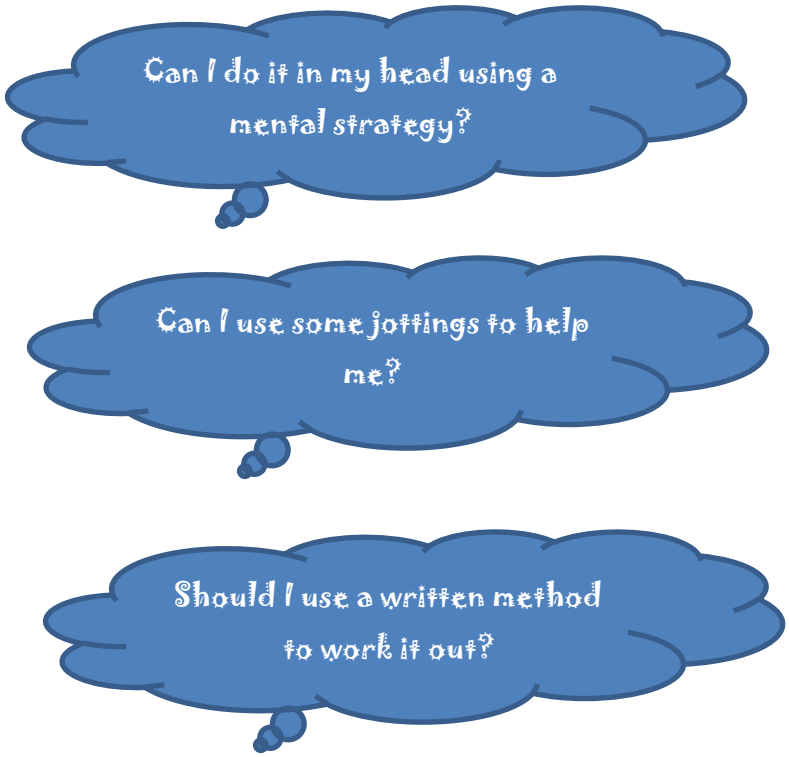
This policy is a progressive document and it is vital that the pupils are taught appropriate methods to support them with their understanding of calculation according to the developmental stage they are at. Children must be secure and show they have 'mastered' a stage, before moving on to the next one. This may mean they are working above or below the expectations set out for their age group in the National Curriculum.

**Providing a Context for Calculation:**

It is important that any type of calculation is given a real life context. This combined with a problem-solving approach will help build children's understanding of the purpose of calculation, and help them recognise when to use certain operations and methods when faced with unfamiliar problems.

**Choosing a Calculation Method:**

Each operation is broken into two sections: the steps that lead towards a formal written method; and the range of informal and mental methods that children should be introduced to. Discussion of alternative methods and their decisions for choosing a particular method should form a part of any calculations teaching (see reasoning objectives in the long term plan for age-appropriate expectations). Children need to be taught and encouraged to use the following processes in deciding what approach they will take to a calculation – this will help them select the most appropriate method for the numbers involved:



# Addition

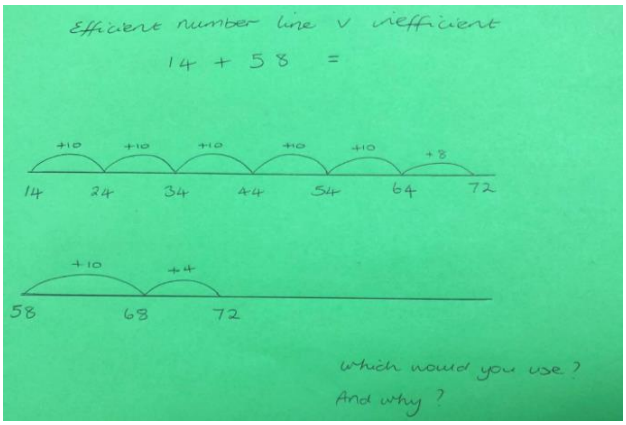
**Key Vocabulary**
Add, addition, more, plus, increase, sum, total, make altogether, double, how many more?, tens boundary, hundreds boundary, partition, inverse, 'carry', decimal places, decimal point.

**Concrete resources:**
100 square
Number lines
Bead strings
Straws
Dienes
Place value cards
Place value dice
Place value counters
Numicon

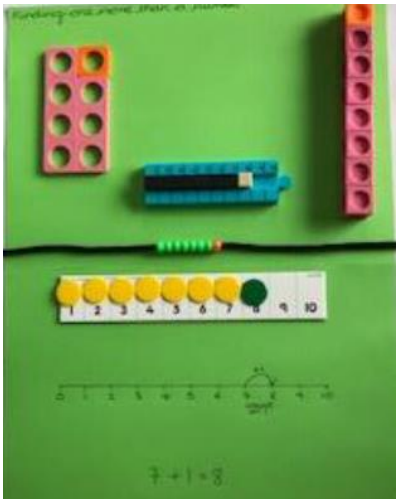
The models & images at each stage are only examples, & a range of different apparatus may be used. However, ways of recording should follow what is set out in this policy to ensure consistency throughout the school.

At all stages, teachers should:

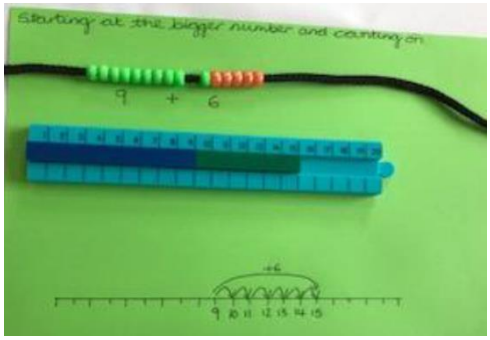
- Develop children's fluency & recall of basic number facts.
- Develop children's fluency in mental calculation & discuss different strategies for solving the same calculation
- Develop children's understanding of the = symbol
- Teach inequality alongside teaching equality
- Use empty box problems
- Move between the concrete and the abstract
- Contextualise the mathematics
- Discuss & explain why processes work
- Make links between the different mathematical operations



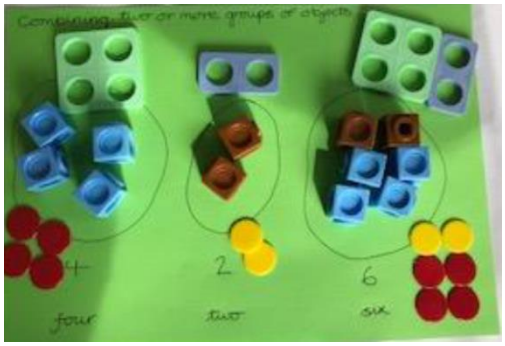
# Early Foundations for Addition



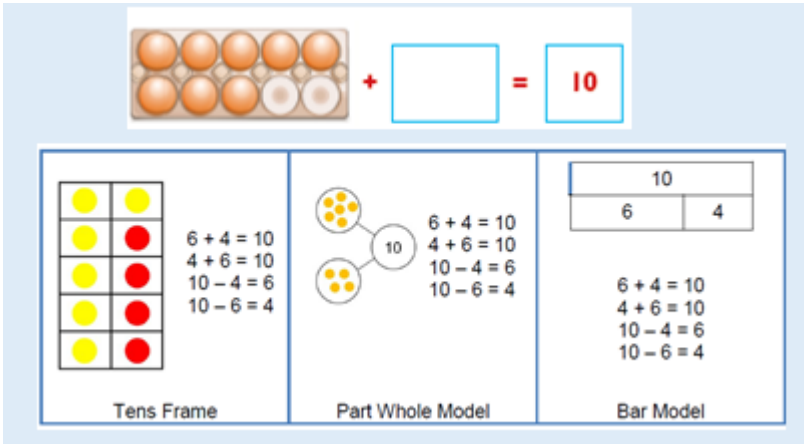
- Know that counting on is a strategy for addition.
- Starting at the bigger number & counting on.



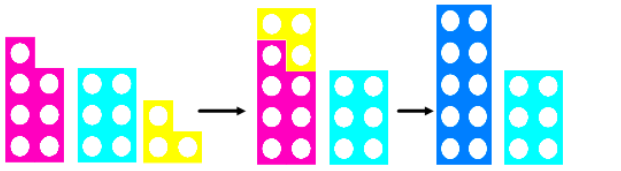

- Relate addition to combining two groups of objects.




- Memorise & reason with number bonds to 10 and 20.

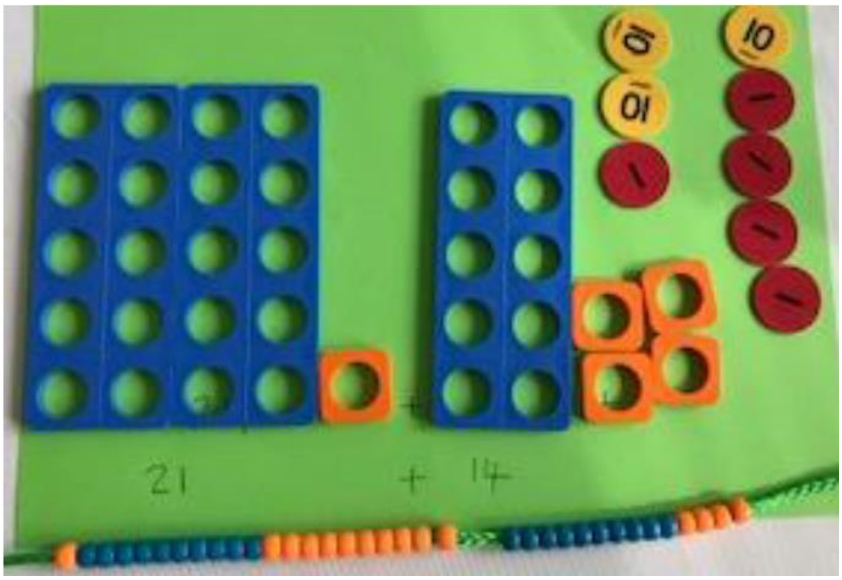


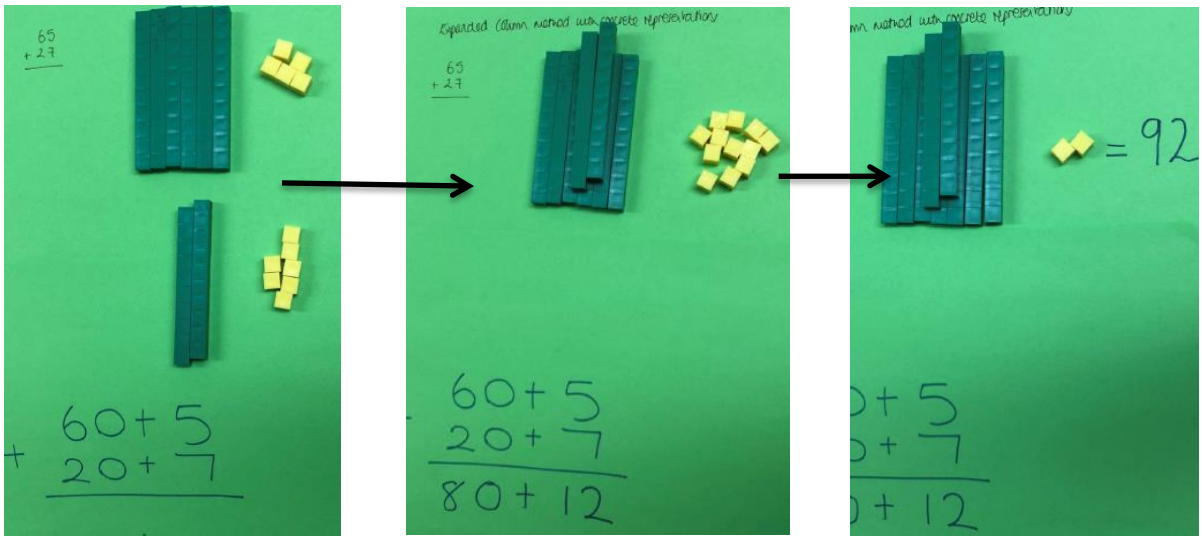

- Through exploration and play, children should understand that addition is commutative: it can be done in any order. *7 + 3 is the same as 3 + 7*
- Add three or more numbers, recognising that they can be added in any order.



# Progression Towards a Formal Written Method

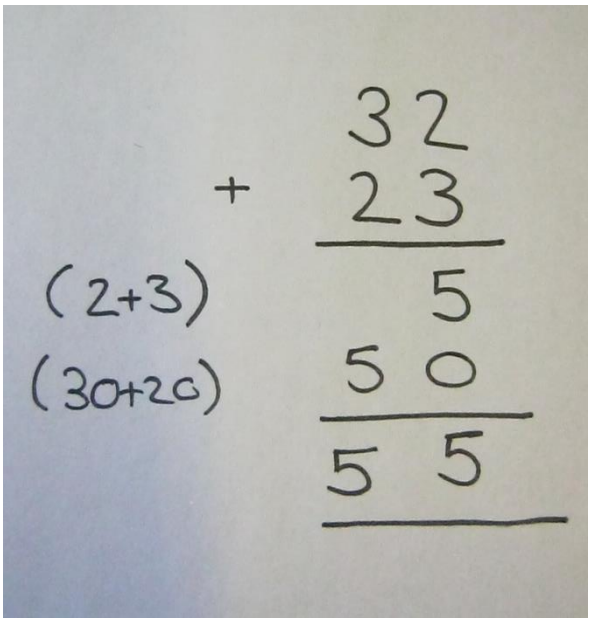
**Step 1: Partition into tens and units & recombine.**



**Step 2: Expanded Column Method with concrete apparatus**

- In preparation for compact column method , teach them to add units first.
- Children should be taught to ensure digits of the same value are in line (square paper can be ued to aid this).

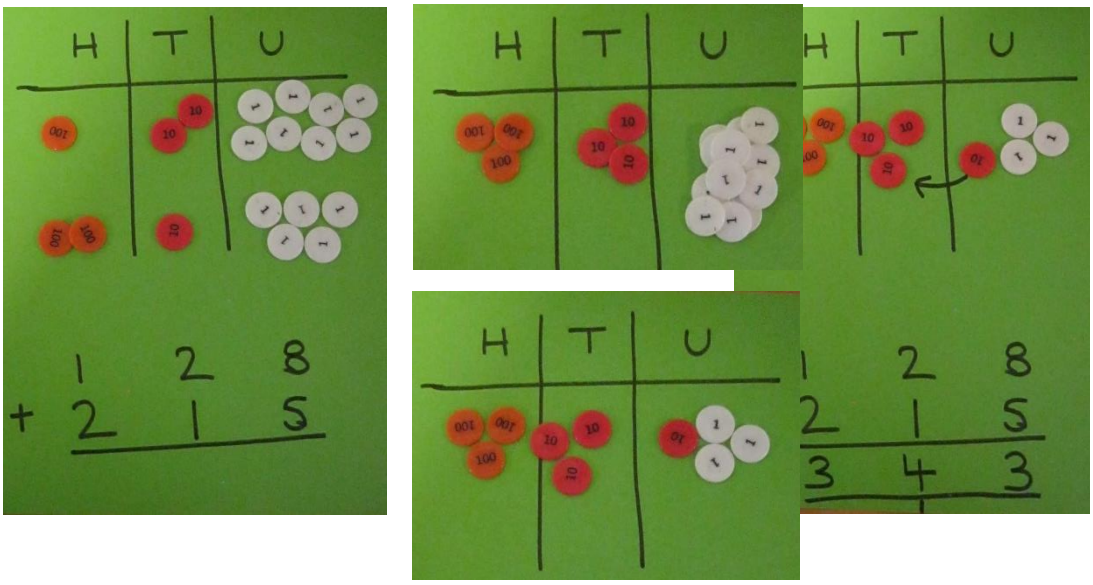



**Step 3: Reduce the expanded method to the compact method, with 'carrying'.**

- The carried digit is as important as any other digit, so should be written in the line and the same size as the other digits.
- The carried digit is recorded at the bottom.

$$\begin{array}{r} 5271 \\ +2357 \\ \hline 7628 \\ \hline 1 \end{array}$$

- This can be modelled using counter and a place value grid:

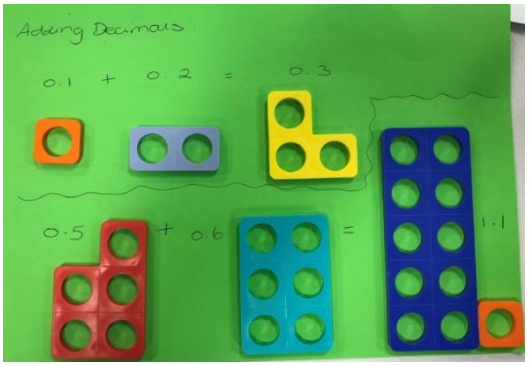


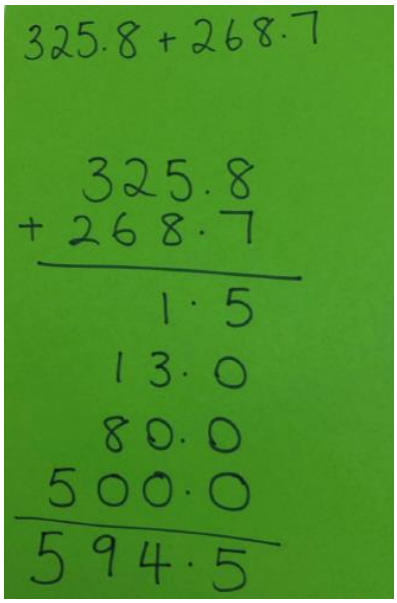

If children experience difficulties, return to the expanded method.

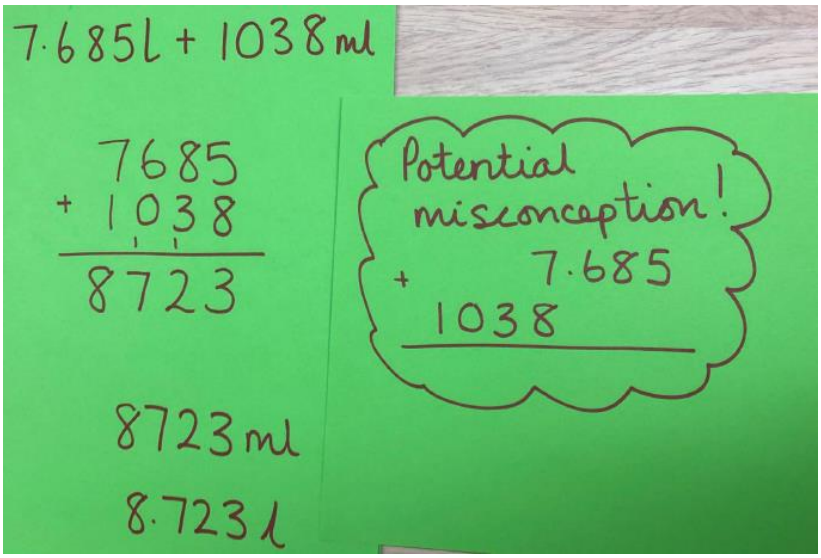
**Using the compact methods with decimals**

- Zero can be used as a place-holder if necessary.
- Children must ensure the decimal point is lined up in the numbers being added and the total. The decimal point must not move.
- Where difficulties are encountered, return to concrete models and the expanded method.

$$\begin{array}{r} 9.080 \\ 59.770 \\ +\ 1.300 \\ \hline 93.511 \\ \hline 2\ 1\ 2 \end{array}$$



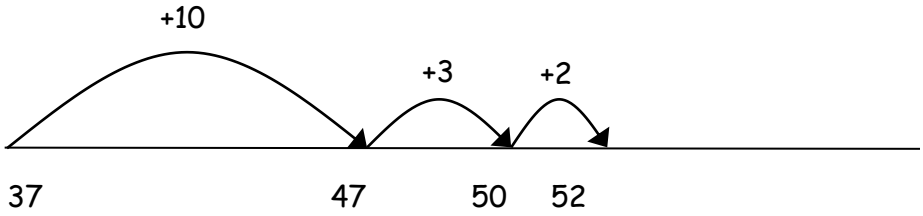
## Informal Methods of Addition

Appropriate informal and mental methods should be taught alongside the above written method, including:

- Counting
- Use of doubles & near doubles
- Number bonds to 10, then 20, then 100, then any multiple of 10 or 100.
- Compensating methods for 9 and numbers near to a multiple of 10 (e.g. adding ten, then subtracting one)
- Partial partitioning (e.g. 27 + 37, add the thirty to make 57, then add another 7)
- Bridging multiples of 10 (e.g. 37 +8, add three to make 40, then another five to make 45)

The above informal methods should be modelled with Numicon, number lines, number squares, bead strings, counters, place value apparatus, cards & counters etc.

### Partial Partitioning & Bridging: 37 + 15 = 52

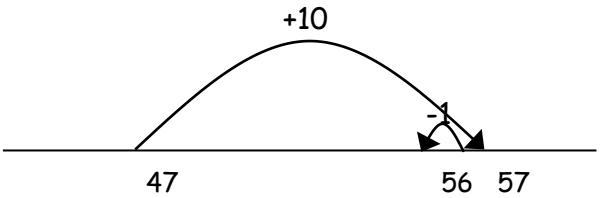



### Compensating:

37 + 29 = 66 (+30, then -1)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 6 | | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

47 + 9 = 56



# Subtraction

**Vocabulary**
Subtract, take away, minus, decrease, count on, count back, how many are left?, how many less?, half, halve, inverse, difference between, fewer,

**Concrete resources:**
100 square
Number lines
Bead strings
Straws
Dienes
Counting stick
Place value cards, dice & counters
Numicon

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At all stages, teachers should:

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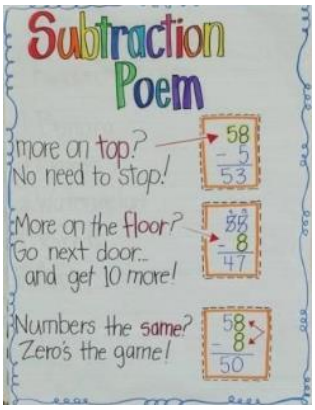
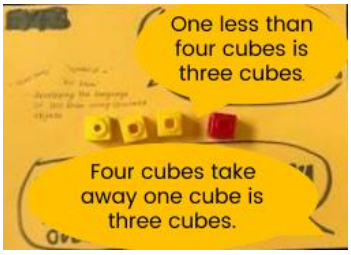
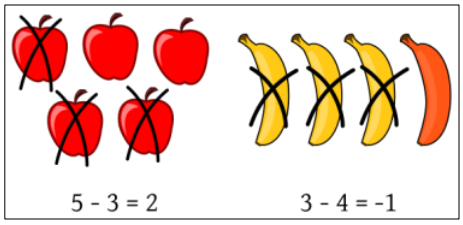


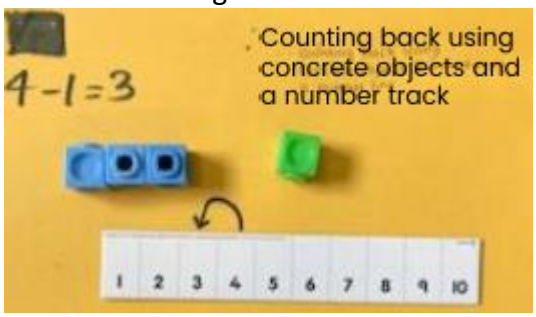
# Early Foundations for Subtraction

- Relate subtraction to taking away concrete objects & role play.

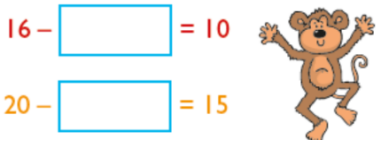


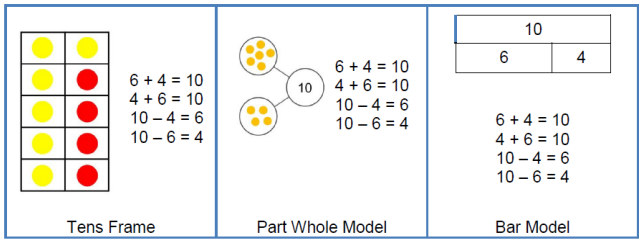




- Know that counting back is a strategy for subtraction.
- Count backwards along a number line to take away.

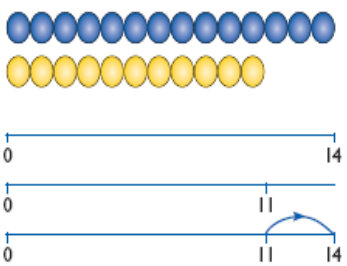



- Memorise & reason with number bonds to 10 and 20.

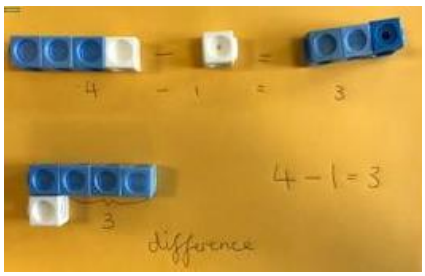



- Begin to relate subtraction & difference.
  NB: Counting-on should only be used when the language is *'find the difference'*, *'difference between'* or *'distance between.'*



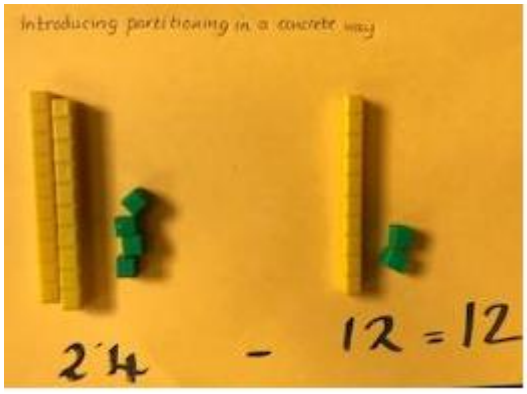

The difference between 11 and 14 is 3.
14 – 11 = 3
11 + ☐ = 14




- Children must explore and understand that subtraction is not commutative: unlike addition, it cannot be done in any order. *4 – 3 is not the same as 3 - 4*

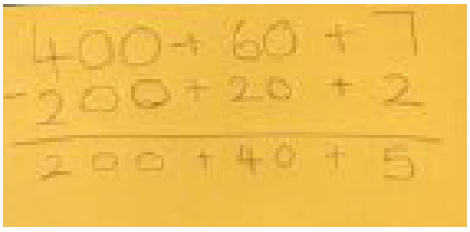
# Progression Towards a Formal Written Method

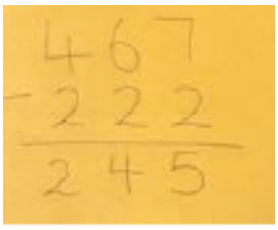
**Step 1: Introduce partitioning in a concrete way (no exchange).**





798 – 235 = 563

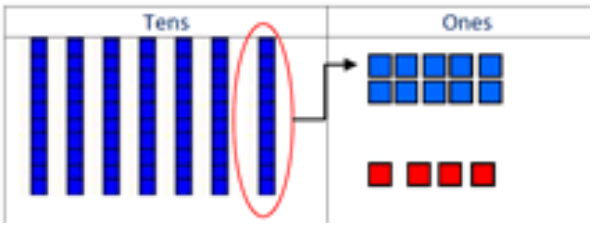


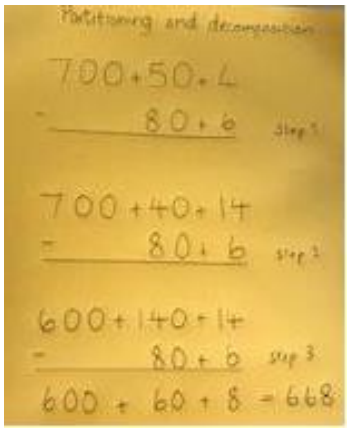



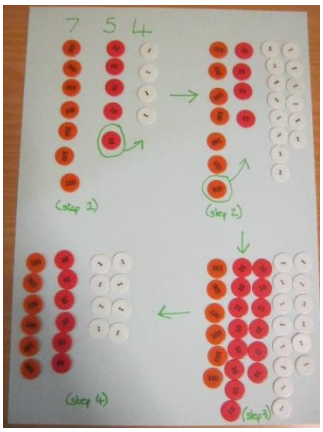

- When recording, children should be taught to ensure digits of the same value are in line (square paper can be ued to aid this).
- In preparation for the compact method, children should start with the smallest digit.
- Children must always start from the top and take the number below away.

**Step 2: Exchange a ten for 10 ones or a hundred for 10 tens.**

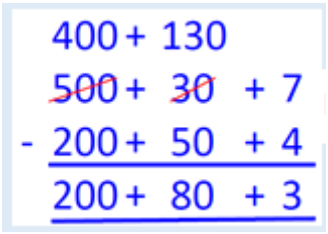
- The need to exchange is established through concrete modelling.
  *"We need to take away 7, but we only have 4. What can we do? We can exhcnage a ten for ten ones."*



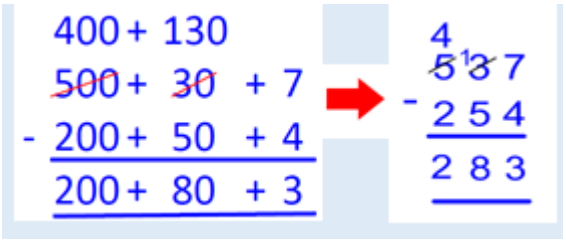



**Step 3: Record using an expanded decomposition method.**




- This should be modelled with concrete apparatus (see step 2).
- By this stage children should be confident in starting from the top number and taking away the bottom number. However, if children begin to 'reverse' the calculation, take them back to concrete modelling to reinforce this.

**Step 4: Reduce the expanded method to the compact decomposition method:**

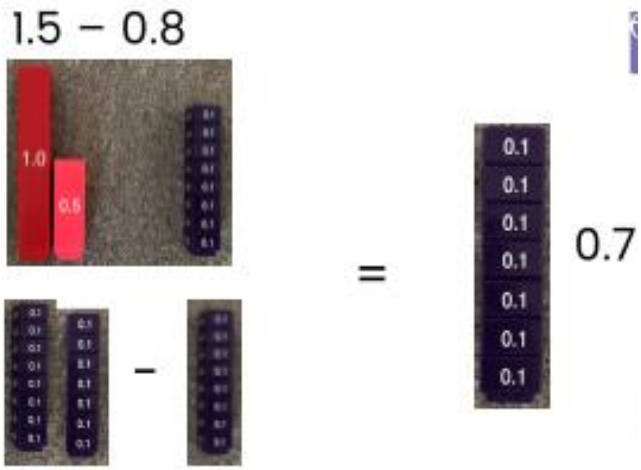



- The exchanged ten or hundred is just as important as any other number. Therefore, it should be written as clear and as large as any other number, and placed at the **top** of the column which has been adjusted.
- Where children experience difficulties, they should return to the expanded method and concrete models.

**Using the decomposition method with decimals**

- Zero is added as a place holder.
- Children must ensure the decimal point is lined up in the numbers being subtracted and the total. The decimal point must not move.

Where difficulties are encountered, return to concrete models and the expanded method.



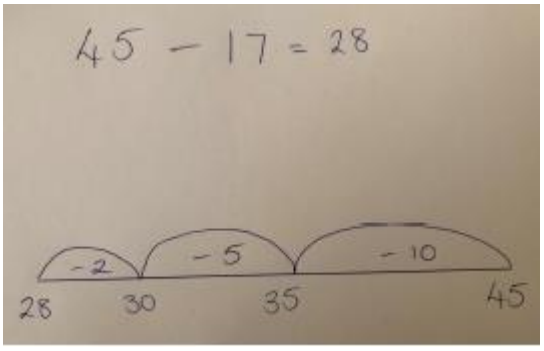

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | ~~1~~ | ~~10~~ | 15 | . | ~~3~~ 4 | 11 | 9 kg |
| – | | 36 | . | 0 | 8 | 0 | kg |
| | | 69 | . | 3 | 3 | 9 | kg |

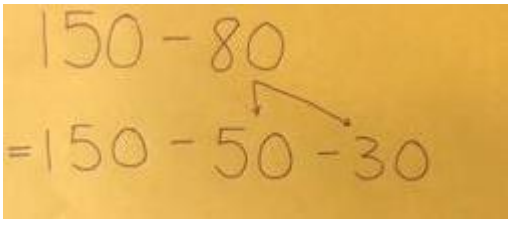
# Informal Methods of Subtraction

Appropriate informal and mental methods should be taught alongside the above written method, including:

- Counting backwards (including use of number lines)
- Number bonds to 10, then 20, then 100, then any multiple of 10 or 100.
- Working out the difference – especially when it is a small difference (e.g. 201 – 198)
- Use of inverse relationship with addition (If I know 4+6=10, then I know 10-6)
- Compensating methods for 9 and numbers near to a multiple of 10 (e.g. subtracting one hundred, then adding one back on)
- Partial partitioning (e.g. 1357 - 230, subtract 200, then subtract 30)
- Bridging multiples of 10 (e.g. 63 - 5, subtract 3 to get to 6, then another 2 so you've subtracted 5)

The above informal methods should be modelled with Numicon, number lines, number squares, bead strings, counters, place value apparatus, cards & counters etc.

# Multiplication

**Key Vocabulary**
Lots of, groups of, times, product, multiply, multiple of, repeated addition, array, row, column, double, commutative, square, factor, integer, grid method

**Concrete resources:**
Place value counters
Dienes
Place value charts
Arrays
Multiplication squares
100 square
Number lines & Blank number lines
Counting stick
Numicon

The models & images at each stage are only examples, & a range of different apparatus may be used. However, ways of recording should follow what is set out in this policy to ensure consistency throughout the school.

At all stages, teachers should:

- Develop children's fluency & recall of basic number facts.
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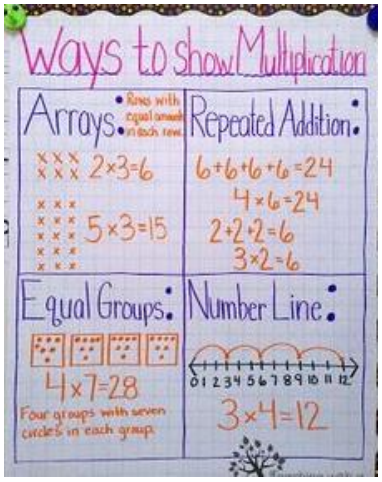
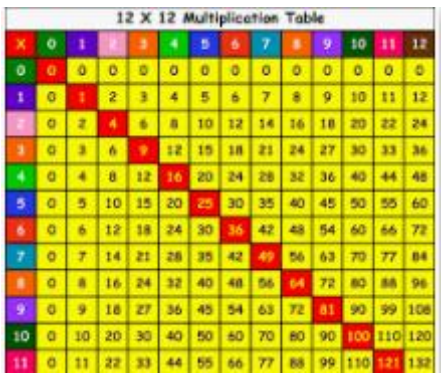
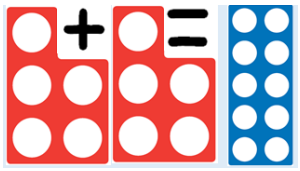
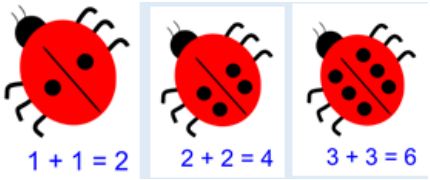


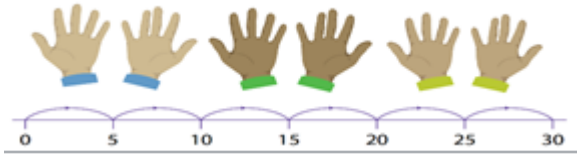
# Early Foundations for Multiplication

- Use pictorial representations and concrete resources to double numbers to 10.






- Use concrete sources, role play, stories and songs to begin counting in twos, fives and tens.




- Use pictorial representations of groups.


- Make & draw arrays.
- Understand multiplication as repeated addition – use concrete objects to support understanding.
- Explore and recognise the commutative nature of multiplication – it can be done in any order: 3 x 6 is the same as 6 x 3.

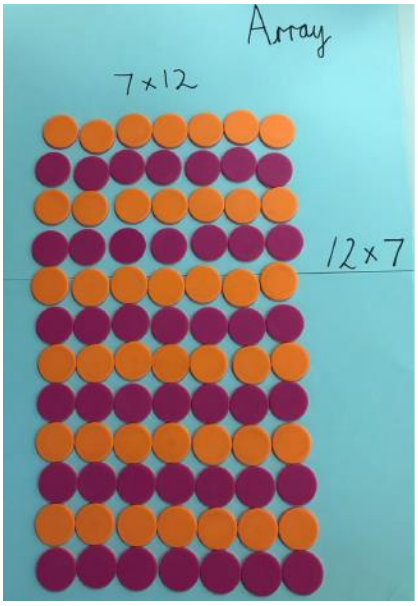
5 + 5 + 5
or
3 x 5

3 + 3 + 3 + 3 + 3 = 15

5 + 5 + 5 = 15

3 x 5 = 15

5 x 3 = 15

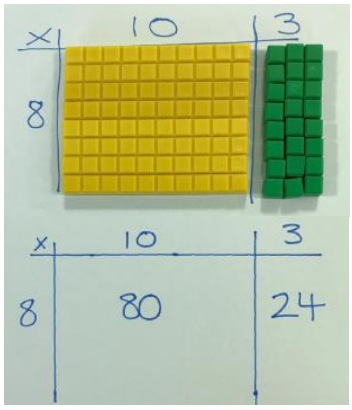


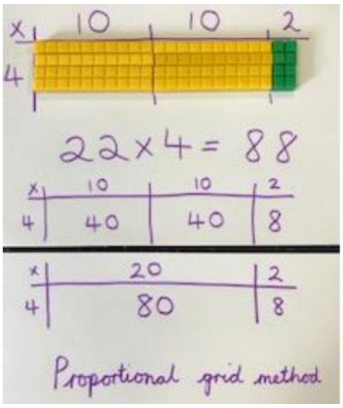
# Progression Towards a Formal Written Method

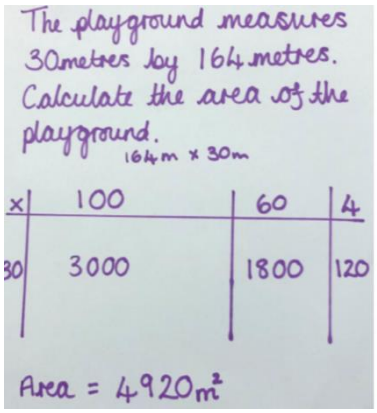
**Pre-requisite: A confident recall of times tables is required to use these methods effectively – see appendix A for the progression in & teaching of times tables.**

**Step 1: Model multiplication using arrays and the grid method.**

Proportional grids and apparatus should be used to reinforce place value.



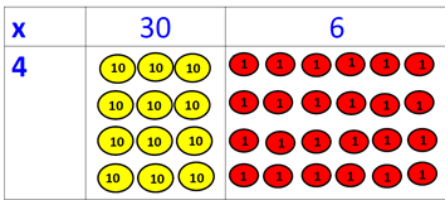



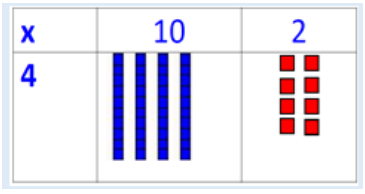

Children should be taught to start from the smallest digits when multiplying in order to prepare for moving to column methods.

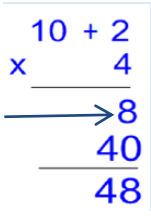
Links can be made to area.

**Step 2: Use place value apparatus to model multiplication by partitioning and link this to the expanded column method.**

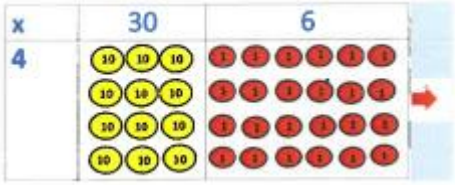


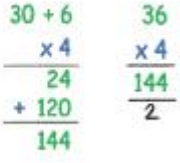

or




**Step 3: Reduce the expanded column method to compact short multiplication.**



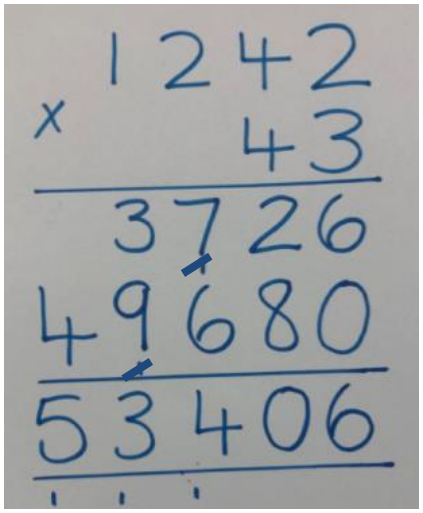


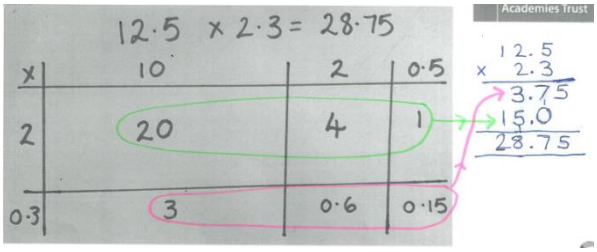


The carried digit is as important as any other digit, so should be written in the line and the same size as the other digits.

The carried digit is recorded at the bottom.

**Step 4: Use compact short multiplication to multiply up to 4-digits by 2-digits and whole numbers by decimals.**




When multiplying by two digit numbers, children should record the carried digit under the next column, and cross it through once it has been added on. This avoids confusion and numbers been added more than once.

When multiplying by a tens number, a zero is added in the ones column 'as a place holder'.

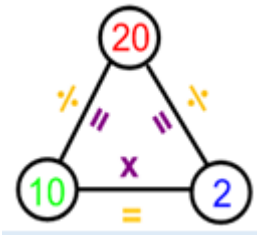
Where difficulties occur, children should go back to modelling with practical apparatus and the links between the grid method and the column methods.

## Informal Methods of Multiplication

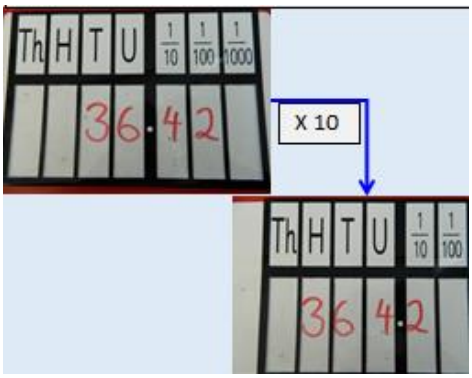
Appropriate informal and mental methods should be taught alongside the above written method, including:

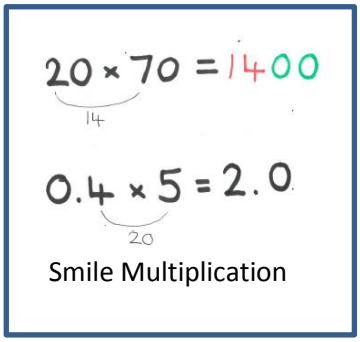
- Recall of times tables (see appendix A)
- Fact families & the link between multiplication & division facts
- Repeated addition & counting in multiples
- Counting on from known facts (e.g. If I know seven fives are thirty-five, then I can count on seven to find 6 x 7) and counting back (e.g. 10 x 6 = 60, so 9 x 6 = 60-6 = 54).
- Doubling & halving known facts (e.g. 2x is double; 4 x is double and double again; or 12 x 3 = double 6 x 3; or 5 x7 is half of 10 x 7)
- Use of place value to multiply by 10s, 100s, 1000s etc. NB: Be aware of the misconception of "adding a zero". While this appears to work with whole numbers, it does not work for decimals. The digits are moving one place to the left and the zero fills the empty place. This can be modelled with place value charts (see below).
- Smile multiplication using tables knowledge and place value

The above informal methods should be modelled with Numicon, number lines, number squares, bead strings, counters, place value apparatus, cards & counters etc.

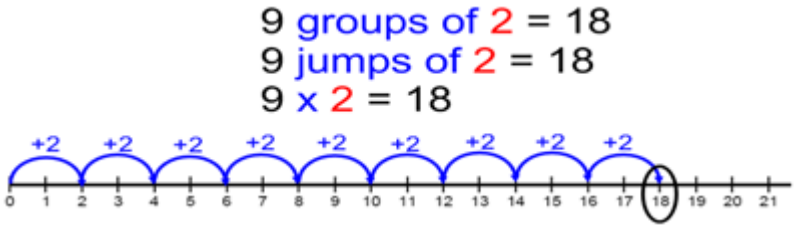







Smile Multiplication



# Division

**Key Vocabulary**

share, share equally, equal groups of, lots of, array, divide, divided by, division, grouping, left over, inverse, remainder, quotient, divisor, factor, prime number, prime factor, composite number (non-prime)

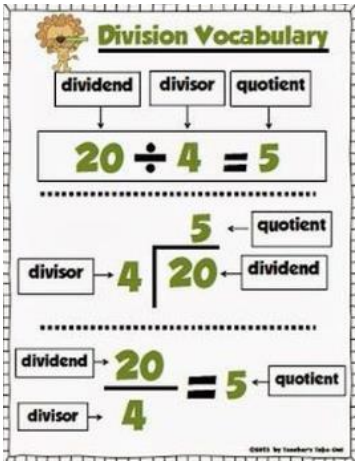
**Concrete resources:**

Arrays
Multiplication squares
100 square
Number lines
Blank number lines
Counting stick
Place value apparatus
Numicon

The models & images at each stage are only examples, & a range of different apparatus may be used. However, ways of recording should follow what is set out in this policy to ensure consistency throughout the school.

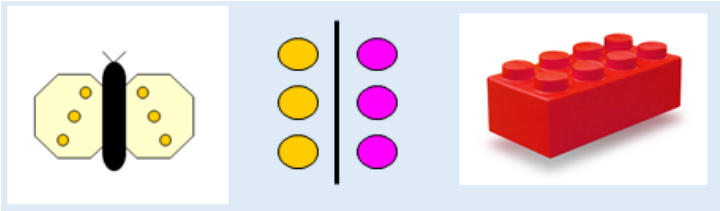
At all stages, teachers should:

- Develop children's fluency & recall of basic number facts.
- Develop children's fluency in mental calculation & discuss different strategies for solving the same calculation
- Develop children's understanding of the = symbol
- Teach inequality alongside teaching equality
- Use empty box problems
- Move between the concrete and the abstract
- Contextualise the mathematics
- Discuss & explain why processes work
- Make links between the different mathematical operations

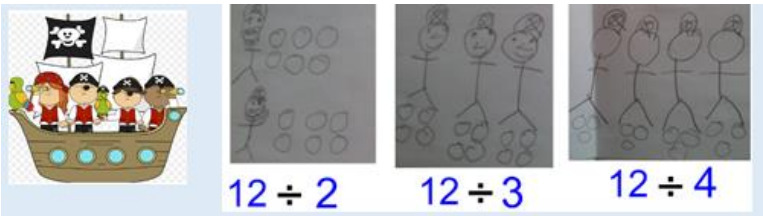


# Early Foundations for Division

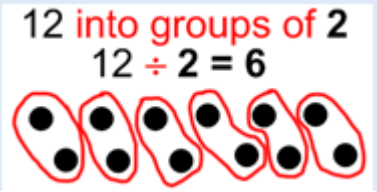
- Use pictorial representations and concrete resources to halve numbers.
- Begin to share quantities using practical resources, role play, stories and songs.



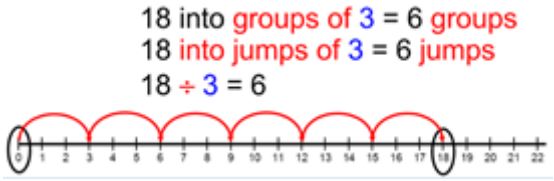
- Understand division as sharing using concrete resources and pictures.




- Understand division as grouping using concrete resources and pictures.
  Model division as grouping on a number line.








- Reinforce division through the use of arrays.

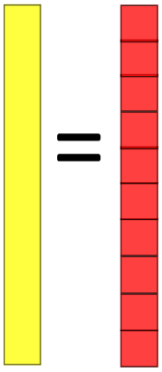

- Recognise division as the inverse of multiplication and link to times tables.
- Remember to develop connections between fractions and division *e.g. 1/3 of 18 is the same as 18 ÷ 3 = 6.*

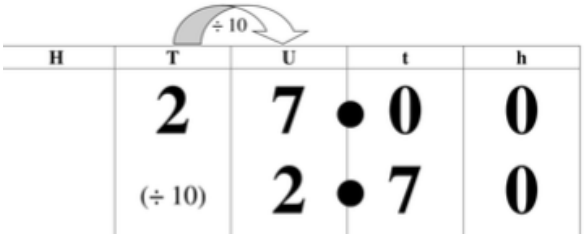
# Progression Towards a Formal Written Method

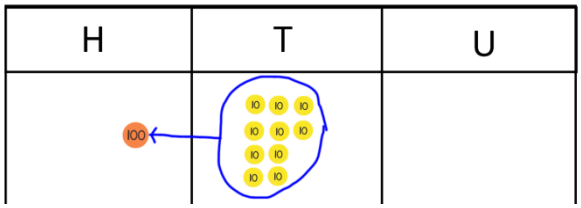
**Pre-requisite: A confident recall of times tables is required to use these methods effectively – see appendix A for the progression in & teaching of times tables.**

**Children should use arrays, concrete resources and number lines to understand the link with repeated addition, times table knowledge and grouping.**

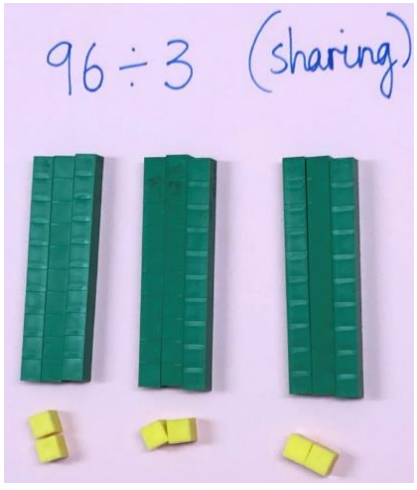
**Step 1: Use place value to multiply and divide by 10 and know that 10 ones can be exchanged for 1 ten, 10 tens for 1 hundred, 10 tenths for 1 whole etc & vice versa.**

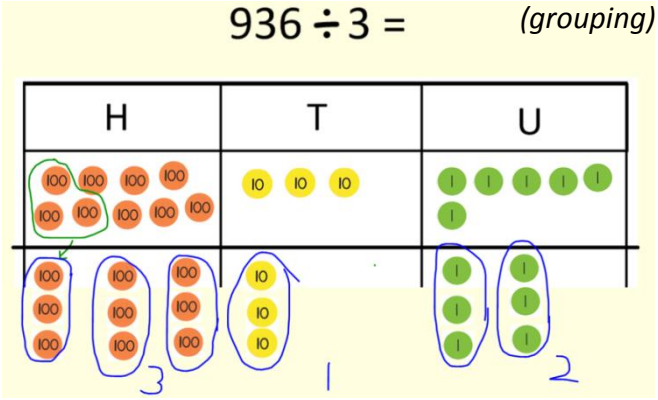




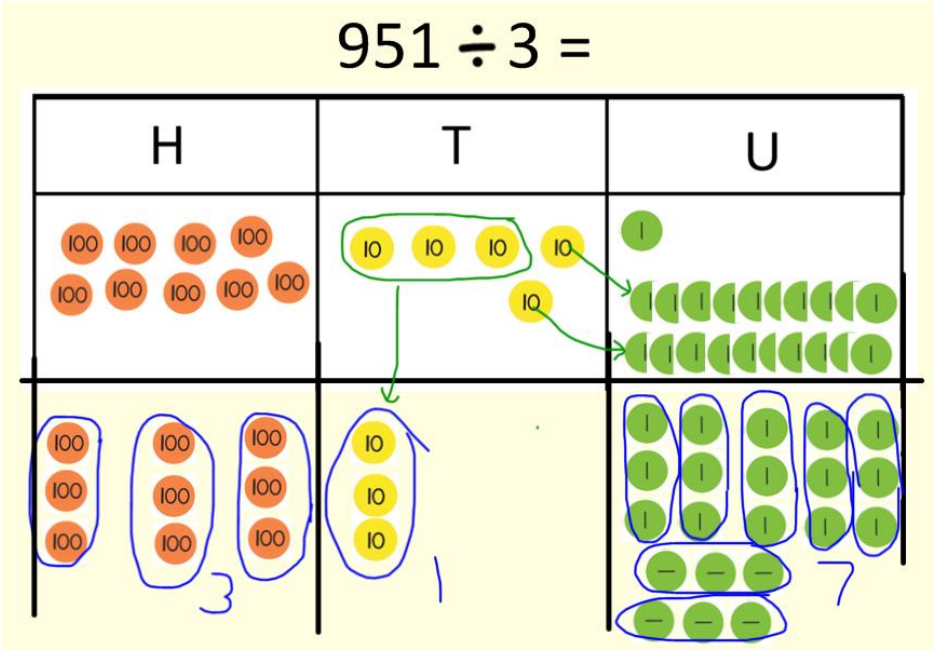




**Step 2: Use practical apparatus to model partitioning of numbers in order to divide (no exchange).**



**Step 3: Model exchange using place value apparatus & link this to the formal method of recording short division.**




$$\begin{array}{r} 3\,1\,7 \\ 3\overline{)\,9\,5^{2}1} \end{array}$$

The remainder is written in front of the digit in the next column to show it has been exchanged for ten times as many.

**Step 5: Understand remainders can be remainders, fractions and decimals.**

$$218 \div 8 =$$

$$\begin{array}{r} 2\,7\ \text{r}\ 2 \\ 8\overline{)\,2^{2}1^{5}8} \end{array}$$

- Whole number remainder = 27 r 2
- Fraction remainder = $27\frac{2}{8} = 27\frac{1}{4}$
- Decimal remainder = $27\frac{1}{4} = 27\frac{25}{100} = 27.25$

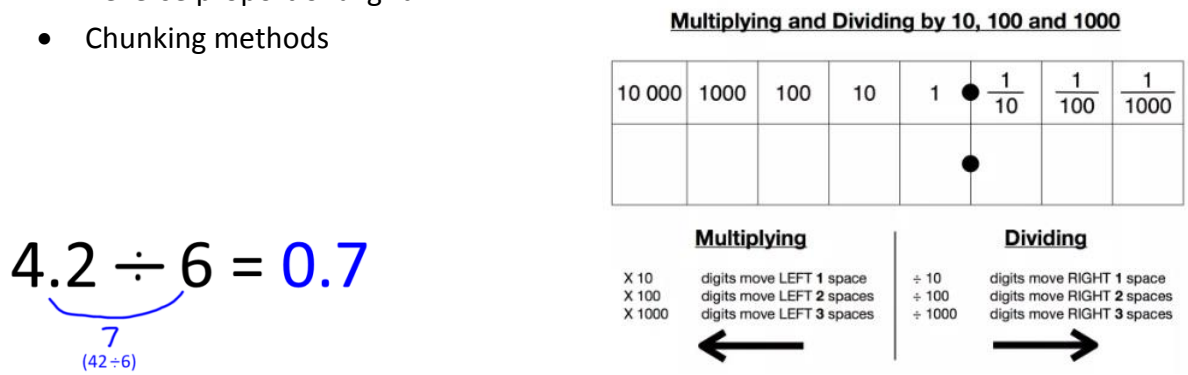
The formal short division method can be continued into decimal places in order to calculate decimals equivalents of fractions. Children must ensure they remember the decimal point in their answer.

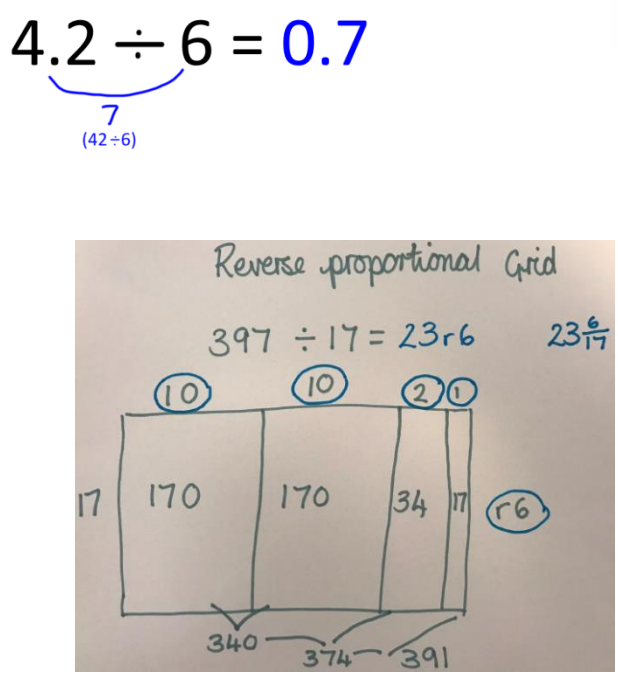
$$\begin{array}{r} 0\,2\,7\,.\,2\,5 \\ 8\overline{)\,2^{2}1^{5}8\,.^{2}0^{4}0} \end{array}$$

# Informal Methods of Division

Appropriate informal and mental methods should be taught alongside the above written method, including:

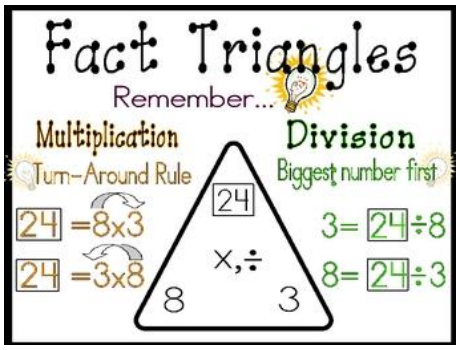
- Fact families & the link between multiplication & division facts
- Repeated addition & counting in multiples
- Repeated subtraction
- Use of place value to divide by 10s, 100s, 1000s etc. NB: Be aware of the misconception of "taking off the zero". While this appears to work with whole numbers, it does not work for decimals. The digits are moving one place to the right. This can be modelled with place value charts (see below).
- Smile division using tables knowledge and place value.
- Reverse proportional grid
- Chunking methods

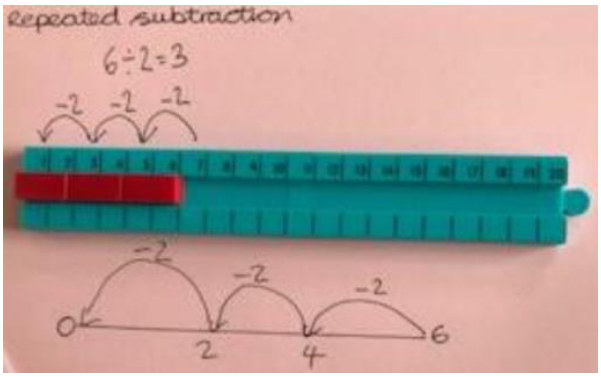


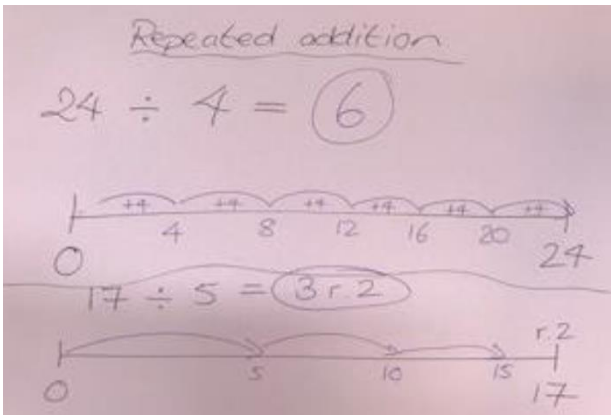









## Appendix A: Progression in Multiplication Tables

This document sets out the order that times tables should be taught in. While the expected year group each table should be mastered is given, this will not be appropriate to all to children. Children should be taught the tables in the designated order, even if this means going back to prior year groups before working at their chronological age.

How do we teach times tables?

Tables need to be taught and frequently practised not just tested. Strategies include:

- Counting forwards & backwards in multiples: start with multiples displayed, then slowly cover more & more until children can recite without them on display.
- Counting sticks: forwards & backwards; ensure you say the fact (3 x 2) as well as the multiple (6) to make the link between the two; link multiplication & division facts when counting; slowly cover up or miss out more multiples; once children are more confident 'hop' around rather than always counting in order.
- Teach children to count on from known facts (e.g. you know 5x, so count on to find 6x, 7x etc).
- Teach children to use related facts (e.g. the 4x table is double the 2x table).
- Reciting rhymes & songs (& include actions).
- Explore patterns in times tables.
- Link the working out of times table facts to repeated addition, counting and arrays.
- Filling in blank multiplication squares & grids.
- Commutative law: if you 2 x 3, then you also know 3 x 2.
- Waldorf multiplication flowers (see appendix)

| NB: Children should be able to *quickly* recall all facts in a random order at each stage before progressing. | Year Group Expectations |
|---|---|
| First:<br>• x 1<br>• x 10 | Year 1 |
| Second:<br>• x 5<br>• x2 | Year 2 |
| Then:<br>• x 4<br>*Double the 2x table.* | Year 3 |
| • x 3<br>• x 6<br>*These can be linked: once children know x3, they can double it to learn x6. X6 can also be worked out by counting on from the known x5.*<br>• x 9<br>*Teach counting back from 10x & the 'finger trick'.* | Year 3 |
| • x 8<br>*Double the x4 table.* | Year 4 |
| Finally:<br>• x 7<br>*While there are no easy tips & tricks for this one, if they know all the other tables then they will be able to work out all of the sevens – just learn 7x7.* | Year 4 |
| Now:<br>• Children should be able to answer quick-fire questions on any times table, mixed up and in any order.<br>• They should be able to apply their knowledge to multiply larger numbers using place value (e.g. if you know 3x6, you can work out 30x6, 300x60 etc.) and partitioning (e.g. 14x7 = (10x7) + (4x7) ). | Years 5 & 6 |

NB:

If children know tables up to 10x then they are able to calculate any other from 11 upwards by partitioning. However, when learning tables it is good practice to learn up to 12x to boost children's confidence in quick recall.


Waldorf Flowers

Children start this activity by drawing the centre of the flower, in which they write a number between 2 and 12. They then draw 12 petals around the centre, with each petal containing the numbers 1 through 12. The last step is to draw another set of 12 petals which contain the centre number multiplied by each petal in the inner circle.

## Appendix B: Progression in Counting

Counting is a fundamental part of maths and should be included in every lesson. Counting should always be modelled both forwards and backwards, and be supported by suitable models & images. The expected counting in each year group is set out below, but this should be followed flexibly. All year groups may need to revisit prior counting skills and children should be extended when appropriate.

| |
|---|
| Reception<br>• Count forwards to 10 and back again.<br>• Count on from any 1 digit number.<br>• Count up to 20 and back again, being very clear with the 'teen' pronunciation.<br>• Count along with the numbers displayed. |
| Year 1<br>• Count up to 50 and back again.<br>• Count up to 100 and back again.<br>• Count along with the numbers displayed and beginning to count along a scale without the numbers marked. |
| Year 2<br>• Count up in steps of 10 forwards and backwards.<br>• Count in steps of 2 forwards and backwards.<br>• Count in steps of 5 forwards and backwards.<br>• Count forwards and backwards beyond 100.<br>• Count in halves & wholes. |
| Year 3<br>• Continue counting in steps of 2, 5 and 10 and related multiples of 10 & 100.<br>• Count in steps of 25 forwards and backwards.<br>• Count in multiples of any single digit forwards and backwards.<br>• Count in multiples of 10 forwards & backwards.<br>• Count forwards and backwards beyond 1000.<br>• Count in halves, quarters and wholes. |
| Year 4<br>• Count in multiples of any single digit & related multiples of 10 (link to multiplication)<br>• Continue counting in steps of 10, 20, 25 and 50; 100, 200, 250 & 500.<br>• Count forwards and backwards into negative numbers.<br>• Count in halves and steps of 0.5 (link to counting in 5s/50s).<br>• Count in quarters and steps of 0.25 (link to counting in 25s).<br>• Count in tenths and steps of 0.1. |
| Year 5<br>• Continue counting in multiples of any single digit and related tenth (e.g. 3 & 0.3).<br>• Count in different steps into negative numbers.<br>• Count in tenths and hundredths, using fractions, decimal and percentages.<br>• Count in halves & quarters, using fractions, decimals & percentages.<br>• Count in fraction steps, using mixed numbers & improper fractions. |
| Year 6<br>• Continue counting in fraction steps, including improper fractions, mixed numbers and equivalent fractions, decimals & percentages.<br>• Count in fifths, and equivalent decimals & percentages (relate to counting in 2/20s).<br>• Count in decimals steps with up to three decimal places. |

Models & images to support counting

- Number lines, tracks, washing lines etc (blank and numbered)
- Number squares (blank and numbered)
- Real objects, money, numicon
- Rulers, metre sticks, scales etc.
- Counting stick
- Bead string
- Fraction plates, fraction Smartboard images